WORKING paper

BANQUE DE FRANCE
EUROSYSTÈME

The EA-BDF Model and Government Spending Multipliers in a Monetary Union*

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September 2022, WP #883

Abstract

We develop in this paper a new two-country model of the euro area (EA-BDF), based on the large-scale FR-BDF model of France and a new medium-scale block of the rest of the euro area (STREAM). This new block follows an approach close to FR-BDF, being a semi-structural model with the same type of adjustment costs and that we can use with different types of expectations. Both countries of EA-BDF share a common endogenous monetary policy and, thanks to our multi-country setup, we can deal with both symmetric and asymmetric shocks. Our illustrations about the effects of a government spending shock in a monetary union deliver two key results, which are robust whatever the type of expectations. First, by studying symmetric and asymmetric shocks on government spending, kept constant for 2 years, we find that, at this 2-year horizon, trade spillovers would compensate monetary policy spillovers within the euro area. Second, we also find, in the case of a symmetric shock, that the government spending multiplier is smaller under a monetary policy rule based on price-level targeting than on inflation targeting.

Keywords: Semi-Structural Modeling, Expectations, Monetary and Fiscal Policies.

JEL classification: C54, E37

* We are grateful to Pascal Jacquinot for his comments and discussing our work at Banque de France seminar. We thank Y. Kalantzis and J.-F. Ouvrard for all their feedbacks and L. Giuliani for his excellent research assistance. We also thank H. Le Bihan and participants of an internal workshop of Banque de France for their comments. The views expressed are those of the authors and do not necessarily reflect those of the Banque de France.

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NON-TECHNICAL SUMMARY

We build a new two-country model of the euro area (EA-BDF), based on the large-scale FR-BDF model of France and a new medium-scale block of the rest of the euro area (STREAM). STREAM follows a similar approach to FR-BDF. It is also a semi-structural model inspired by the FRB/US approach: it uses the Polynominal Adjustment Costs (PAC) framework and includes explicit expectations that can be either VAR-based (VBE), model-consistent (MCE) or hybrid (HYB). We estimate this model equation-by-equation under VAR-based expectations, using the same structural VAR model (E-SAT) as in FR-BDF, but extended with REA variables.

We have made several simplifications in STREAM compared with FR-BDF. First, on the supply side, potential output is exogenous and a NK price Phillips curve based on the unemployment gap determines inflation (GDP price deflator). We do not explicitly model the labor market and the price-wage loop: an Okun's law relates unemployment and output gaps. Second, on the demand side, we simply relate nominal income of households to nominal GDP with a reduced-form error-correction equation. Then, we relate household consumption to permanent income and to interest rate, with a role for current demand in the short run. Main drivers of total investment are demand and the expected real cost of capital, based on the sovereign long rate. As in FR-BDF, the government uses a fiscal rule on social transfers for stabilizing its budget balance-to-GDP ratio toward a level consistent with a target of the debt-to-GDP ratio.

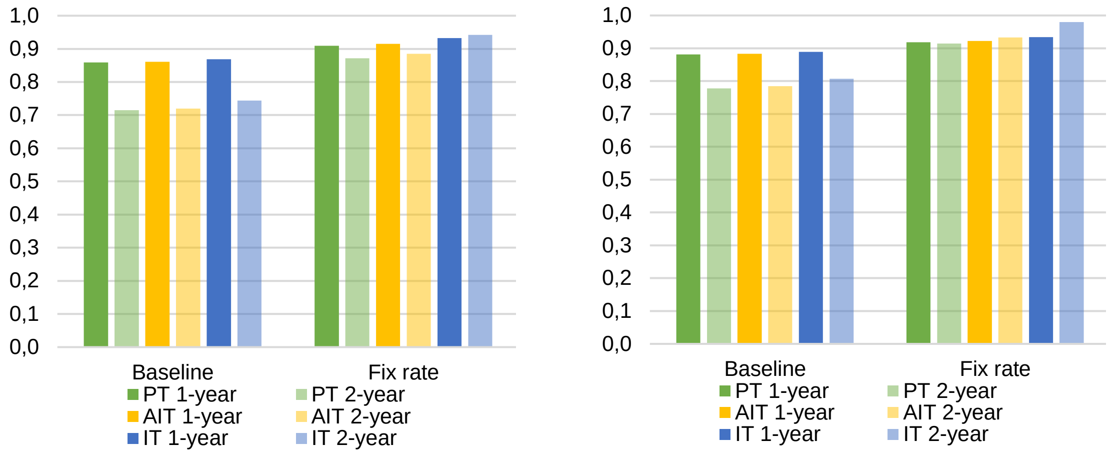
In order to capture trade spillovers between France and REA, we make less simplification with respect to the trade block. We model both consolidated and internal REA exports and imports (volume and deflators). Consolidated exports and imports depend on foreign/internal demand and relative price, through error-correction models. We relate internal imports (volume and deflators) to REA demand and prices; we assume internal exports equal to internal imports. Finally, the euro effective exchange rate and euro/dollar exchange rate equations (real UIP conditions) are common to both models and the REA term-structure is similar to that of France, applied to a weighted average of the four biggest REA countries' sovereign bond rates.

Given its two-region structure, the EA-BDF model allows studying various shocks, including asymmetric shocks at the EA level, under different types of expectations and endogenous monetary policy response. First, we are able to simulate EA-wide shocks with an endogenous reaction of monetary policy and its transmission to long-term interest rate and nominal effective exchange rate. As a result, we can consider different types of monetary policy rules (inflation, price-level or average-inflation targeting rules) and their respective stabilization properties in response to shocks. Second, the ability to switch from VAR-based to model-consistent or hybrid expectations is a strength of our model, in particular with respect to questions related to monetary-fiscal interactions and alternative monetary policy rules. Third, having a two-country model allows us to simulate both symmetric and asymmetric shocks and evaluate spillover effects from trade and monetary policy response.

In this paper, we study the effects of a persistent government spending shocks in a monetary union and obtain two main results, which are robust whatever the type of expectations. First, by studying symmetric and asymmetric shocks for both regions, we find that positive trade spillovers would compensate negative monetary policy spillovers in France, at a 2-year horizon. Second, we study monetary-fiscal interactions under alternative monetary policy rules: inflation-targeting, price-level targeting and average-inflation targeting) and alternative expectations (model-consistent or hybrid). We find that fiscal multipliers are always higher when monetary policy is transitorily constrained (or accommodative) because of its effects on expected inflation, real cost of capital and the exchange rate. Finally, when monetary policy is transitorily constrained, fiscal multipliers are lower under the price-level targeting rule compared to the inflation-targeting rule, because of the future monetary policy tightening, which is required for the price-level reversal.

Government consumption multipliers in the euro area under different monetary policy rules and expectation formation mechanisms

a) Under model-consistent expectations b) Under hybrid expectations



Note: in the model-consistent version of the model, all agents (both financial and non-financial) are forward-looking while in the hybrid version, only financial agents are forward looking while non-financial agents are backward looking.

Le modèle EA-BDF et les multiplicateurs budgétaires dans une union monétaire

RÉSUMÉ

Nous développons dans ce papier un nouveau modèle à deux régions de la zone euro (EA-BDF), basé sur le modèle à grande échelle FR-BDF pour la France et un nouveau bloc de taille moyenne pour le reste de la zone euro (STREAM). Ce nouveau bloc est construit selon l'approche de FR-BDF, c'est-à-dire comme un modèle semi-structurel avec le même type de coûts d'ajustement et que nous pouvons utiliser avec différents types d'anticipations. Les deux régions de EA-BDF partagent une politique monétaire endogène commune et, grâce à notre configuration multi-régions, nous pouvons traiter des chocs symétriques et asymétriques. Nos illustrations des effets d'un choc de dépenses publiques dans une union monétaire fournissent deux résultats clés, qui sont robustes quel que soit le type d'anticipations. Premièrement, en étudiant des chocs symétriques et asymétriques sur les dépenses publiques, maintenues constantes pendant 2 ans, nous trouvons qu'à cet horizon de 2 ans, les effets de débordements commerciaux compenseraient ceux de la politique monétaire au sein de la zone euro. Deuxièmement, dans le cas d'un choc symétrique, nous trouvons également que le multiplicateur des dépenses publiques est plus faible sous une règle de politique monétaire basée sur un ciblage du niveau des prix que sur un ciblage de l'inflation.

Mots-clés : modélisation semi-structurelle, anticipations, politiques monétaire et budgétaire

Les Documents de travail reflètent les idées personnelles de leurs auteurs et n'expriment pas nécessairement la position de la Banque de France. Ils sont disponibles sur publications.banque-france.fr

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1 Introduction

In order to run projections and some of their policy analysis at a country level, large national central banks (NCBs) of the Eurosystem use semi-structural country models.[1] Because of the exogeneity of the external environment in projections, these models follow a small open economy setup, the endogeneity of trade spillovers being taken into account through trade consistency exercices (Hubrich & Karlsson, 2010). For counter-factual simulations, NCBs can also quantify trade spillovers of shocks using elasticities of Eurosystem country models. Still, this framework does not allow to analyze the response to shocks with an endogenous response of monetary policy. Even if the short-run interest rate of ECB is constrained by the effective lower bound in the current juncture, its future endogenous response should matter for the short-run response to current shocks. Hence, there is a need for a rest-of-euro-area extension of current country models, which would allow to endogenize trade spillovers and the response of monetary policy.

In this paper, we develop such a rest-of-euro-area extension in the case of France and we illustrate its usage with a policy experiment, the assessment of the effects of a fiscal stimulus in a monetary union, the euro area. This extension is a medium-scale macroeconomic model for the rest of the euro area (REA), which we call the Semi-sTructural Rest of Euro Area Model (STREAM). Connected to our large-scale semi-structural macroeconomic model for France (FR-BDF) and to an exogenous block of the rest of the world, STREAM allows us to build a two-country semi-structural macroeconomic model for the euro area (EA-BDF). Complementary to elasticities of NCB models, EA-BDF allows us to study various shocks, including asymmetric shocks under different types of expectations and with an endogenous response of monetary policy.

STREAM follows a similar approach to FR-BDF. It is also a semi-structural model inspired by the FRB/US approach: it uses the Polynominal Adjustment Costs (PAC) framework and includes explicit expectations that can be either VAR-based (VBE), model-consistent (MCE) or hybrid (HYB). This approach is used more and more for projection purposes, as it allows to combine the flexibility of traditional macro-econometric models and to have an explicit role for expectations as in DSGE models. We estimate this model equation-by-equation under VAR-based expectations, using the structural VAR as in FR-BDF (E-SAT) extended with REA variables. We have made several simplifications in STREAM compared with FR-BDF. First, on the supply side, potential output is exogenous and a New Keynesian price Phillips curve based on the unemployment gap determines inflation (GDP price deflator). We do not explicitly model the labor market and the price-wage loop: an Okun's law relates unemployment and output gaps. Second, on the demand side, we simply relate nominal income of households to nominal GDP with a reduced-form error-correction equation. Then, we relate household consumption to permanent income and to interest rate, with a role for current demand in the short run. Main drivers of total investment are demand and the expected real cost of capital, based on the sovereign long rate. As in FR-BDF, the government uses a fiscal rule on social transfers for stabilizing its budget balance-to-GDP ratio toward a level consistent

[1] See for example the Makro model of the Bundesbank, FR-BDF of Banque de France, BIQM of the Banca d'Italia or MTBE of the Banco de España.

with a target of the debt-to-GDP ratio. In order to capture trade spillovers between France and REA, we make less simplification with respect to the trade block. We model both consolidated and internal EA exports and imports (volume and deflators). Consolidated exports and imports depend on foreign/internal demand and relative price, through error-correction models. We relate internal imports (volume and deflators) to REA demand and prices; we assume internal exports equal to internal imports. Finally, the euro effective exchange rate and euro/dollar exchange rate equations (real UIP conditions) are common to both models and the REA term-structure is similar to that of France, applied to a weighted average of the four biggest REA countries' sovereign bond rates. We checked that basic model elasticities (BMEs) of STREAM are generally quite close to those implied by the Eurosystem National Central Banks' BMEs for the REA.

The EA-BDF model, which connects FR-BDF to STREAM, allows studying various shocks, including asymmetric shocks at the EA level, under different types of expectations and endogenous monetary policy response. First, we are able to simulate EA-wide shocks with an endogenous reaction of monetary policy and its transmission to long-term interest rate and nominal effective exchange rate. Second, our approach allows to assess how the type of expectations (VAR-based, model-consistent or hybrid) might matter for model-based assessments. Third, having a two-country model allows us to simulate both symmetric and asymmetric shocks and evaluate trade and monetary-policy spillovers.

Within the multi-country modelling literature, the approach we follow for EA-BDF has some similarities with the ECB-MC model currently in development at ECB, aiming to be the multi-country extension of ECB-BASE of Angelini et al. (2019): our EA-BDF model shares some common ingredients with respect to the way to model expectations and adjustment costs, but our REA block does not need to be as detailed as their country blocks, as we do not use it for projection purposes. Our approach for the REA block has also some common features with the one followed for a typical block of the flexible system of global models (FSGM) of Hunt et al. (2015), whose EUROMOD module contains 11 blocks for main euro area countries plus 13 other blocks. Indeed, our REA block is also a semi-structural model, has also a medium-scale size and also allows for forward-looking expectations. Still, our setup is more flexible as it allows to switch the type of expectations between VAR-based and model-consistent expectations for each of these equations. Finally, we should also mention the model EAGLE of Gomes et al. (2012), a Eurosystem model which distinguishes as our model two members of the euro area sharing a common monetary policy, but which is less comparable to our model for two reasons: first, it follows a structural and not a semi-structural approach; second, it is less focused on data fit, as its parameters are calibrated and not estimated.[2]

In this paper, we illustrate the usage of our multi-country model of the euro area by studying the effects of a government spending stimulus in a monetary union and their sensitivity to the type of expectations. First, we focus on the multipliers of a symmetric shock on government spending, kept constant for 2 years and faded out afterward. For both countries and whatever the type of

[2] In the same vein, with a structure close to EAGLE, Castelletti Font et al. (2018) designed FREAM, a model of France within the euro area, which they use for comparing the effects of tax cuts targeted toward labor or capital.

expectations, we find an output multiplier close to one in the short run, which masks contradictory forces: on one side, we have some crowding-in effects of household consumption and total investment thanks to the rich frictions of the model; on the other side, due to the tightening of monetary policy, these effects are dampened by the rise of the real cost of capital and the appreciation of the real exchange rate generates some crowding-out effects of net trade. Second, we focus on the multipliers and spillovers of an asymmetric shock on government spending originating in France. Compared to the symmetric case, it appears, again whatever the type of expectations, that the output multiplier of France is similar, i.e. that the output spillovers in REA are close to zero at the two-year horizon of the shock. These results reflect that trade spillovers compensate monetary policy spillovers at this horizon. Third, we study how the multipliers of a symmetric shock on government spending depend on monetary policy rules. The main result is that, whatever the type of expectations, when the monetary policy is constrained to keep its interest rate constant during the period of the shock, the output multiplier would be smaller with monetary policy based on price level targeting than on inflation targeting.

In our first application related to fiscal multipliers within the euro area, we study a topic already covered by the model-based literature, but we have two main contributions: (i) we run the experiments with a semi-structural model which puts a stronger emphasis on empirical fit than structural models and (ii) we study the robustness of such results with respect to the type of expectations of agents.[3] A useful benchmark within the model-based literature on government spending mulitipliers is the paper of Kilponen et al. (2019), which compares such multipliers in normal times and under a fix interest rate for the euro area as a whole and in euro area countries with several structural models of these regions. In the case of a temporary cut, they find for the euro area a multiplier equal to 0.9 in normal times. For an asymmetric shock in France, they find a multiplier of 0.8 in normal times. Our results are similar for France, but we get a larger multiplier for the euro area in normal times (around 1) because, within our semi-structural approach, we find a smaller sensitivity of consumption to the monetary policy tightening stemming from the demand hike and corresponding inflation pressures.

Concerning the spillovers of an uncoordinated government spending shock, there is a puzzling divergence between model-based and empirical assessments reported in the literature. Recent model-based literature that has focused on the size of fiscal spillovers in the monetary union report that within a liquidity trap, the inter-regional spillovers are positive and large, while they are slightly negative in normal times.[4] The latter is due to the fact that monetary policy spillovers are larger than positive spillovers related to trade. Empirical assessments however generally find substantial positive spillovers in normal times. This is for example the case in Beetsma & Giuliodori (2011) and in the more recent work of Alloza et al. (2020). If we focus on their SVAR results, available

[3] First, semi-structural models based on the PAC framework seem to dampen monetary policy effects and to be less prone to the forward guidance puzzle than standard DSGE models, as shown for example by Chung (2015) through a comparison of FRB/US with EDO and Smets-Wouters DSGE models. Second, the ability to switch between VAR-based, model-consistent and hybrid expectations in our semi-structural model offers a complementary perspective to DSGEs, which generally assume either rational expectations or model-consistent expectations under perfect foresight.

[4] See for example Blanchard et al. (2017) and in't Veld (2017).

both for origin spillovers (output response in other EA countries of a stimulus originating in one country) and destination spillovers (output response in a destination country of a stimulus taking place in all other EA countries), Alloza et al. (2020) find for France that the peaks of the origin and destination spillover would be around 0.1 and 0.2.[5] In an analysis based on a multi-country DSGE model, in the case of government consumption shock, they show that they can recover small positive origin spillovers, but that destination spillovers would be slightly negative within this model. With our semi-structural model, we find spillovers slightly positive and close to zero whatever their type (destination/origin) and the type of expectations: when the shock hits France the GDP of REA would increase only by around 0.02%; when the shock hits REA countries, the GDP of France would increase by around 0.10%. Hence, if we do not fully solve the puzzle, we still obtain results closer to the empirical evidence. Compared to structural models, two features of our approach might play a role in this outcome: first, the lower sensitivity of consumption to the real interest rate already discussed above might reduce in our model the negative spillover generated by monetary policy; second, the large sensitivity of imports to internal demand in the short run might reinforce the positive trade spillovers.

As Pedersen et al. (2021), we study in our last application the interaction between monetary and fiscal policies under different types of expectations. Regarding the evaluation of alternative monetary policy rules (like price-level targeting or average inflation targeting), the semi-structural approach is fairly novel and offers a complementary perspective to analysis based on structural models.[6] In this area, recent papers by Bernanke et al. (2019) and Bernanke (2020) specifically use semi-structural models, like the FRB/US model, to assess the stabilization properties of alternative monetary policy rules and their ability to avoid Effective Lower Bound (ELB) episodes. In contrast with them, we rather focus on monetary-fiscal interactions and the size of fiscal multipliers under alternative monetary policy rules. Our work also links with the literature on state-dependent fiscal multipliers, when monetary policy is passive (Leeper et al., 2017) or when the ELB is binding (Christiano et al., 2011). Still, to our knowledge, Pedersen et al. (2021) is the only other paper than ours to explore the sensitivity of fiscal multipliers to alternative monetary policy rules. Compared to results obtained with structural models of this paper, we find a higher persistence of government spending multipliers and a smaller difference between multipliers across monetary regimes (active monetary policy or fix interest rate).[7]

The rest of the paper is structured as follows. The next section provides details about the specification and estimation of STREAM and explains how we build the EA-BDF model based on FR-BDF and STREAM. Section 4 is devoted to the impulse responses of this two-country model. Section 5 analyzes the transmission of government spending shocks through the lens of this model. Section 6 concludes.

[5] They obtain this result for samples starting either in early eighties and ending either in 2016. Hence, a large share of their sample concerns normal times (before the Great Financial Crisis). Their sample also includes a large period earlier than the launch of the EMU, where exchange rate adjustments might have amplified fiscal spillovers.

[6] See the seminal papers by Svensson (1999) and Eggertsson & Woodford (2003).

[7] We also find a smaller difference between multipliers across monetary regimes than in results reported in Kilponen et al. (2019).

2 The Semi-sTructural Rest of Euro Area Model (STREAM)

This section describes STREAM in detail. After describing the estimation approach and the notations we use, we proceed block by block for presenting the version of STREAM based on VAR expectations. In a last subsection, we also explain how we modify this version of STREAM for building a version based on model-consistent expectations.

2.1 Data and estimation approach

In order to be consistent with the estimation of FR-BDF, we estimate the model using quarterly national accounts on a sample going from 1995-Q1 to 2017-Q4.[8] We generally compute data of the rest of the euro area by chained-price substraction of EA aggregated data minus French data. Based again on French and EA data, we use more complex formulas for the computation of some data related to external trade (see appendix A.1 for details): first, we need to decompose gross trade flows of the rest REA region into consolidated and internal flows, in order to take into account properly the small openness degree of the consolidated REA; second, we need to compute intra/extra foreign demand and competitor's prices of REA, some key variables for the connection of the REA region to France and to the exogenous rest of the world.

Many of the equations describing short-run dynamics include expectation terms which require estimating a model to compute these terms. The model we apply is an extended version of the satellite model of expectations (E-SAT) we use for computing VAR-based expectations of FR-BDF (see subsection 2.3 for details), where we also model expectations regarding REA variables. The estimation process is as follows: (i) we estimate this extended version of E-SAT using Bayesian methods and (ii) we estimate the rest of the model equation-by-equation using iterative OLS for PAC equations and simple OLS for other equations.

With respect to the extended version of E-SAT, we estimate its core equations with Bayesian methods, while we estimate all auxiliary equations needed for expectations formation with OLS.

With respect to PAC equations, we follow two steps. First, all long-run equations describing targets are estimated with simple OLS. Second, following the methodology of FRB/US, the short-run equations are all estimated with iterative OLS. The iterative estimation of the short run equations of the model is based on OLS: an initial guess is made on the PAC coefficients of the equation, which can be used to compute a discounted sequence of expectations using the extended E-SAT, which is in turn used as an observable in the estimation of the PAC coefficients. Given these new estimates, the expectations sequence can be recomputed, the PAC coefficients re-estimated and so forth until convergence.

The estimation of equations following the PAC framework requires an assumption to be made regarding the discount factor β appearing in the intertemporal cost minimization problem of each agent. We follow the approach of FRB/US and calibrate this number to be 0.98 in all main blocks

[8] We use the vintage available in the database of the Macroeconomic Projection Exercise (MPE) of March 2018. Estimation samples can vary depending on: (i) equation specifications, (ii) data availability for variables that are external to QNA and (iii) modellers' judgment.

of the model except household consumption (see section 2.5 for details). This discount factor is consistent with the real rate of return for financial assets of roughly 8% observed in the US during the postwar period (Brayton et al., 1996) and in France during the period 1970-2014, as shown by Garbinti et al. (2017).

2.2 Notation

While the model notation in code follows very closely the standard notation laid out in the System of National Accounts (United Nations, 2009), we prefer here to use shorter notations in order to present more compact formulas, as in Lemoine et al. (2019).

In order to distinguish variables of different geographical areas, we include them within subscripts, e.g. with $x_{REA,t}$ for the variable x of REA. In the specific case of France, we do not include any geographical indication within subscripts as in the FR-BDF model.

Our notation introduces a number of operators that are particularly common in STREAM. The first is the expectations operator, which we denote by $\mathrm{PV}\,(x)_{t|t-k}$ where x_t is a model variable. The subscript describes the timing of information: the first component t refers to the date when the expectation is constructed, while the second component $t-k$ refers to the information set available for the construction. The second operator is the gap, by which we mean the deviation of x_t from its long run trend $\bar{x}_t$. This gap is denoted by $\widehat{x}_t$, i.e. $\widehat{x}_t = x_t - \bar{x}_t$.

Furthermore, we follow some typographical conventions in order to simplify our notation. Lowercase letters will be used to denote logarithms – e.g. the logarithm of household consumption C_t will be c_t – or interest rates, e.g. the short rate will be denoted i_t. Other rates or ratios will be denoted with the letter τ.

Finally, Table 2.2.1 presents, for the sake of convenience, the variables and notation for the core variables appearing in the extended version of the expectations satellite model E-SAT, which is transverse across STREAM.

2.3 Extended E-SAT

The satellite model of expectations (E-SAT) of FR-BDF is a small semi-structural model, which can be represented as a VAR and is estimated with Bayesian methods. Its core variables are the output gap and inflation of France and the euro area, as well as the short-term interest rate of the euro area. Expectations are modeled by IS and Phillips curves for France, the same types of equations for the euro area and an inertial Taylor rule for the euro area. We also make the strong simplification that France does not influence the euro area. The core of the model is completed by the long-run anchors of both inflation rates and of the short-term interest rate, which are modeled as simple AR(1) processes. E-SAT can also be applied to compute expectations for variables that are not within the previously described core. In this case, E-SAT has to be augmented with auxiliary equations that describe how the variable(s) of interest are related to the core variables.[9]

[9]For more details about E-SAT, see Lemoine et al. (2019).

Table 2.2.1: E-SAT core variables

Notation	Description
$\widehat{y}_t$	French output gap, in deviation from the long run output
$i_{EA,t}$	Euro area short-run interest rate, 3-month Euribor
$\bar{i}_{EA,t}$	Long-run trend of the euro area short-run interest rate
$\pi_{Q,t}$	Value-added price inflation of French market branches
$\bar{\pi}_{Q,t}$	Long-run trend of the value-added price inflation of French market branches
$\widehat{y}_{EA,t}$	Euro area output gap, in deviation from potential output
$\pi_{EA,t}$	Growth rate of the euro area GDP deflator
$\bar{\pi}_{EA,t}$	Long run trend of the euro area GDP deflator inflation
$\widehat{y}_{REA,t}$	Output gap of the rest of euro area, in deviation from potential output
$\pi_{REA,t}$	Growth rate of the rest of euro area GDP deflator

In order to ease the computation of expectation variables of STREAM, we extend the core of E-SAT with two extra variables of the rest of euro area (REA): the REA output gap $\widehat{y}_{REA,t}$ and the REA inflation rate of the output price $\pi_{REA,t}$. Given that E-SAT already includes such variables for the euro area and for France[10], we model the REA output gap and REA inflation with simple weighted subtractions based on these variables:

$$\widehat{y}_{REA,t} = \frac{\widehat{y}_{EA,t} - \omega_{EA}^{FR}\widehat{y}_t}{1-\omega_{EA}^{FR}} \tag{1}$$

$$\pi_{REA,t} = \frac{\pi_{EA,t} - \omega_{EA}^{FR}\pi_{Q,t}}{1-\omega_{EA}^{FR}} \tag{2}$$

where $\omega_{EA}^{FR} \approx 0.21$ is the average GDP share of France within the euro area.

2.4 Phillips curve

We model the REA GDP deflator price with a New-Keynesian Phillips curve (NK-PC), such that inflation depends on the expected present-value of future unemployment gaps and on the long-run anchor for inflation expectations. Given the small size of STREAM, we made several simplifying assumptions and modeling choices. First, we do not model explicitly the labor market and the wage-prices dynamics, which are absent in the model. We shortcut the wage-prices loop by (i) relating the unemployment gap to output gap using an Okun's law, and then by (ii) directly relating

[10] For simplifying purposes, we assume that expectations of inflation for the GDP price or the value added price of market branches are the same for France.

price-inflation to expected unemployment gap using a NK-PC. Second, the long-run GDP[11] of REA is completely exogenous in STREAM: we abstract from capital accumulation, trend labor force dynamics and trend productivity. In this section, first we start by deriving the NK-PC and presenting its estimation. Second, we describe the way we model expected unemployement gap and its connection to the output gap through an Okun's Law.

Table 2.4.1: Variables used in section 2.4

Notation	**Description**
$\pi_{REA,t}$	GDP price inflation (qoq)
$\widehat{u}_{REA,t}$	Unemployment gap
$\widehat{y}_{REA,t}$	Output gap, in percentage of long-run GDP
$\bar{\pi}_{EA,t}$	Long run trend of the GDP deflator inflation
E-SAT	See Table 2.2.1

New-Keynesian Phillips curve Our approach is similar to the NK Wage Phillips Curve in FR-BDF (Lemoine et al., 2019, see section 4.5.1) but with the simplifying assumption that we do not explicitly model the labor market. We shortcut the wage-price loop by using a NK Phillips curve, in which GDP price inflation depends directly on the expected unemployment gap. We start from the following equation:

$$\pi_{REA,t} - \bar{\pi}_{EA,t} = \beta \mathbb{E}_{t-1}(\pi_{REA,Y,t+1} - \bar{\pi}_{EA,t+1}) + (1-\beta)\kappa \widehat{u}_{REA,t} \tag{3}$$

Solving forward equation (3), ignoring the explosive solution and accounting for the balanced-growth path, we obtain:

$$\pi_{REA,t} = \kappa \mathrm{PV}(\widehat{u}_{REA})_{t|t-1} + \bar{\pi}_{EA,t} \tag{4}$$

where

$$\mathrm{PV}(\widehat{u}_{REA})_{t|t-1} \equiv (1-\beta)\mathbb{E}_{t-1}\sum_{k=0}^{\infty}\beta^k \widehat{u}_{REA,t+k}$$

is the expected present-value of future unemployment gaps. Finally we estimate the following specification with a lag in order to take into account inertia in inflation dynamics:

$$\pi_{REA,t} = \alpha_0 \pi_{REA,Y,t-1} + (1-\alpha_0)\left(\kappa \mathrm{PV}(\widehat{u}_{REA})_{t|t-1} + \bar{\pi}_{EA,t}\right) + \varepsilon_t \tag{5}$$

Estimates of equation (5) are reported in Table 2.4.2 and Figure 2.4.1 shows the dynamic simulation. Results show a low R^2, due to the simplified structure of our Phillips curve, but a negative and strongly significant effect of expected unemployment gap on GDP price inflation. As a result, the

[11]We label it "long-run GDP" since it is the long-run anchor of real GDP. But it could have been labeled "trend GDP" as well, since it is obtained by applying a statistical filter to real GDP.

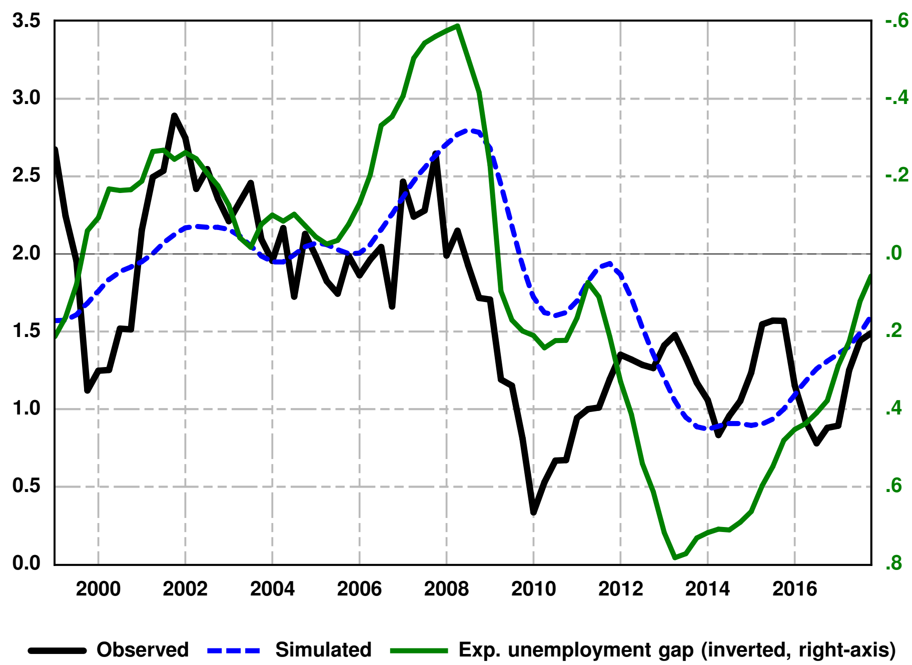
substantial decline in inflation after 2008 significantly reflects positive expected unemployment gap, while the long-run anchor of euro area inflation expectations remains steady. Finally, note that the parameter κ cannot be immediately interpreted as the long-run slope of the Phillips curve (i.e. the long-run effect on inflation of a +1pp shock to the output gap), as it requires to take into account coefficients associated to output gap both in the Okun's law and in the policy function for the expected unemployment gap (cf. *infra*).

Table 2.4.2: Coefficients and standard errors of the Phillips curve

Coefficient	Estimate	s.e.
α_0	0.26	0.18
κ	-0.37	0.09

$R^2 = 0.09$

Figure 2.4.1: Rest of euro area GDP price yoy-inflation and expected unemployment gap, in percentage points



The Okun's law and the expected unemployment gap As explained earlier, as we do not model explicitly the labor market, neither labor demand nor labor supply, we link within STREAM the unemployment gap to the output gap using the following specification of the Okun's law:

$$\widehat{u}_{REA,t} = \beta_0 \widehat{u}_{REA,t-1} + (1-\beta_0)\beta_1 \widehat{y}_{REA,t} + \varepsilon_t \tag{6}$$

where the unemployment gap depends on a lag and on contemporaneous output gap of the rest of euro area. Then, we extend E-SAT using the same Okun's law as an auxiliary equation to get expected unemployment $\mathrm{PV}(\widehat{u}_{REA})_{t|t-1}$. Estimates of equation (6) and coefficients of the policy function for expected unemployment are shown in Table 2.4.3. Concerning the Okun's law, the estimate of its slope β_1 is equal to -0.67, i.e. this law is a bit steeper than the law initially estimated by Okun which had a slope equal to -0.5. However, as the estimate of the inertia β_0 is high (0.92), the transmission of changes of the output gap toward the unemployment gap is slow. Concerning the policy function of the expected unemployment, the coefficient associated to the lagged output gap for REA in the policy function is zero, because the REA output gap is defined as a linear combination of euro area and French output gaps.

In annual terms, the long-run slope of our Phillips curve is obtained from coefficients κ in equation (5), β_1 in equation (6) and coefficients associated to euro area output gap and REA unemployment gap in the policy function for expected unemployment gap :

$$\frac{\partial \pi^4_{REA,t}}{\partial \widehat{y}_{REA,t}} = 4 \cdot (-0.37)\Big((1-\omega^{FR}_{EA}) \cdot (-7.97) \cdot 10^{-2} + (-0.67) \cdot 16.9 \cdot 10^{-2}\Big) \approx 0.27$$

where $\omega^{FR}_{EA} \approx 0.21$ is the share of France in the euro area nominal GDP and $\pi^4_{REA,t}$ is the year-on-year inflation rate. First, we find that STREAM's long-run slope of the Phillips curve is lower than the one of FR-BDF, which is estimated to be 0.45 (Lemoine et al., 2019, p. 48). Our estimate for REA is actually fairly in line with estimates based on reduced-form Phillips curves for the euro area (see Chatelais et al. (2015) for the euro area). Second, given the inertia in the Okun's law, shocks to the output gap gradually transmit to inflation through the Phillips curve and the short-run slope is lower.

2.5 Household consumption

This section describes the determination of household consumption in STREAM. We describe the long run target of household consumption c_t which we derive from an Euler equation and the budget constraint of a representative household. This long run target is based on a permanent income term, with the underlying household primary income determined by an error correction equation. The short run dynamics of consumption are determined with a PAC equation.

Target The target for households' consumption $c^*_{REA,t}$ is based on a permanent income term – as described in (7) – represented by a linear transformation of a standard expectation on the difference between the logarithm of real disposable household income $y_{REA,H,t}$ and the logarithm of real long

Table 2.4.3: Coefficients of the policy function and auxiliary equation for expected unemployment gap and Okun's Law

VAR model	Policy function PV $(\widehat{u}_{REA})_{t\|t-1}$	Auxiliary equation Okun's Law of $\widehat{u}_{REA,t}$
$\widehat{y}_{t-1}$	$0.71 \cdot 10^{-2}$	
$i_{EA,t-1} - \bar{i}_{EA,t-1}$	$26.9 \cdot 10^{-2}$	
$\pi_{t-1} - \bar{\pi}_{t-1}$	$0.56 \cdot 10^{-2}$	
$\widehat{y}_{EA,t-1}$	$-7.97 \cdot 10^{-2}$	
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	$-3.81 \cdot 10^{-2}$	
$\widehat{y}_{REA,t-1}$	0	$-0.67 \cdot F$ [0.14]
$\widehat{u}_{REA,t-1}$	$16.9 \cdot 10^{-2}$	0.92 [0.02]

Note: standard errors in brackets. The F-operator indicates that unemployment gap depends on contemporaneous output gap in the Okun's Law; $R^2 = 0.97$ for the auxiliary equation.

Table 2.5.1: Variables used in section 2.5

Notation	Description
$c_{REA,t}$	Household consumption in REA, volume, in log
$c^*_{REA,t}$	Household consumption target in REA, volume, in log
$y_{REA,H,t}$	Household disposable income in REA, volume, in log
$y_{REA,t}$	REA GDP, volume, in log
$\bar{Y}_{REA,t}$	Long run trend of the volume of REA GDP
$i_{REA,10,t}$	10-yr sovereign bond rate in REA
$r_{REA,10,t}$	Real 10-yr sovereign bond rate in REA
$P_{REA,C,t}$	Deflator, household consumption in REA
$P_{REA,Y,t}$	Deflator, REA GDP
$\tilde{Y}_{REA,H,t}$	Primary income of households in REA
$T_{REA,t}$	Real social transfers in REA
$\overline{\tau}_{REA,YH,\bar{Y},t}$	Hodrick-Prescott trend of logarithm of ratio of household income to long-run trend of volume of GDP in REA
$\hat{\tau}_{REA,YH,\bar{Y},t}$	Hodrick-Prescott cycle of logarithm of ratio of household income to long-run trend of volume of GDP in REA
E-SAT	See Table 2.2.1

Note : the steady-state real household bank lending rate is defined by a spread over the short-term real interest rate and is equal to $(\bar{i} - \bar{\pi}) + \bar{s}_{LH}$ where $\bar{s}_{LH}$ is the term premium.

run GDP $\bar{y}_{REA,t}$.

$$\text{PV}\left(y_{REA,H}\right)_{t|t-1} = \text{PV}\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1} + \bar{y}_{REA,t} \tag{7}$$

Following Campbell & Mankiw (1989), we derive a log-linear consumption equation from the Euler equation and budget constraint of a representative household[12] which determines the consumption target to be a constant share – determined by α_0 – of permanent income.

$$c^*_{REA,t} = \alpha_0 + \text{PV}\left(y_{REA,H}\right)_{t|t-1} + \alpha_1\left(r_{10,REA,t} - \left(\bar{i}_{EA,t} - \bar{\pi}_{EA,t}\right)\right) + \alpha_2 t \tag{8}$$

The equation for the target, (8), also has an additional term that relates consumption to an interest rate gap – between the real yield $r_{10,REA,t}$ of a 10-year bond for the REA[13] and the neutral real short rate, denoted by $\bar{i}_{EA,t} - \bar{\pi}_t$ – which is an attempt to capture long-run effects of interest rates on consumption. In addition, the equation contains a time trend, included in order to capture the upward sloping trend appearing in the ratio of consumption to permanent income, as presented in Figure 2.5.2(a).

The three coefficients of the equation are described in table 2.5.2.

Table 2.5.2: Coefficients and standard errors of the long run equation for household consumption

Coefficient	Estimate	s.e.
α_0	-0.16	0.009
α_1	-0.56	0.60
α_2	0.0004	0.0004

$R^2 = 0.92$

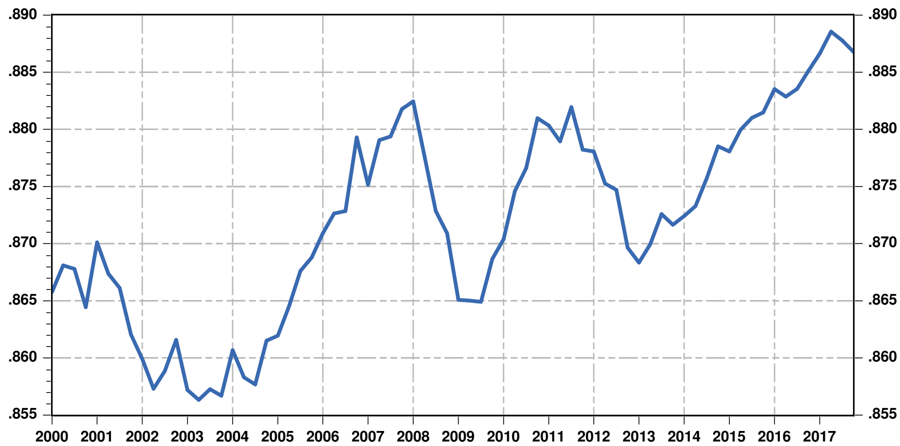
Note that the construction of the permanent income term $\text{PV}\left(y_{REA,H}\right)_{t|t-1}$ is slightly different to the construction of the other expectations in STREAM. In particular, we assume that due to risk aversion and income uncertainty the discount factor applied – i.e. the parameter β – in the equation is somewhat smaller than in the other cases at roughly 0.95. See Reifschneider (1996) for more details on the derivation of this discount factor. The core of the argument rests on the fact that when optimal consumption – solved from a standard household problem – is related to expected uncertain income, this income stream has to be discounted using not just the real rate of interest as in the perfect foresight case, but also a risk adjustment factor that depends on household risk aversion and the variance of the income stream.

Short run equation The short run dynamics of household consumption are described by a first order PAC equation augmented with an additional term – the output gap – intended to represent non-optimizing behavior by rule-of-thumb households ($\Delta\widehat{y}_{REA,t}$). The present value of the target

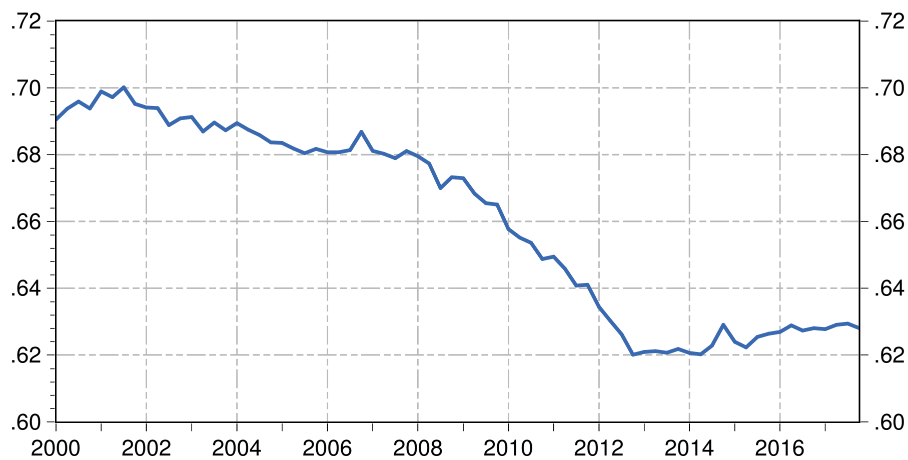
[12] A detailed note is available on request.

[13] See section 2.9 for details on this variable.

Figure 2.5.1: Trends in REA household consumption and real income



(a) Ratio of REA household consumption to REA permanent income



(b) Ratio of REA household real income to REA real long run output

can be split into expectations regarding the components of the target, which includes an expectation regarding permanent income, i.e. an expectation of an expectation.[14] Equation (9) presents this equation and Table 2.5.3 the associated estimated coefficients. Note that α_2 is the corresponding coefficient from (8). We explain below with the construction of expectation variables the construction

[14]Note that as we do not have rational expectations the expectation of an expectation does not equal the expectation.

of the trend income ratio $\overline{\tau}_{REA,YH,\bar{Y},t}$.

$$\begin{aligned}\Delta c_{REA,t} &= \beta_0\left(c^*_{REA,t-1} - c_{REA,t-1}\right) + \beta_1 \Delta c_{REA,t-1} \\ &+ \mathrm{PV}\left(\Delta \widehat{c}^*_{REA}\right)_{t|t-1} \\ &+ \left(1-\beta_1-\beta_{IW}\right)\left(\Delta \bar{y}_{REA,t-1} + \Delta \overline{\tau}_{REA,YH,\bar{Y},t} + \alpha_2\right) \\ &+ \beta_2 \Delta \widehat{y}_{REA,t} + \varepsilon_t\end{aligned} \tag{9}$$

Table 2.5.3: Coefficients and standard errors of the short-run equation for household consumption

Coefficient	**Estimate**	**s.e.**
β_0	0.09	0.04
β_1	-0.04	0.10
β_2	0.32	0.05
β_{IW}	0.65	N/A
$R^2 = 0.69$		

Expectations The expectation term $\mathrm{PV}\left(\Delta \widehat{c}^*_{REA}\right)_{t|t-1}$ has the following decomposition:

$$\begin{aligned}\mathrm{PV}\left(\Delta \widehat{c}^*_{REA}\right)_{t|t-1} &= \mathrm{PV}^2 \Delta\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1} \\ &+ \alpha_1\left[\mathrm{PV}\left(\Delta r_{REA,10}\right)_{t|t-1} - \mathrm{PV}\left(\Delta \bar{i}_{EA}\right)_{t|t-1} - \mathrm{PV}\left(\Delta \bar{\pi}_{EA}\right)_{t|t-1}\right] \\ &+ \beta_{IW}\left[\Delta \bar{y}_{REA,t} + \Delta \overline{\tau}_{REA,YH,\bar{Y},t} + \alpha_2\right]\end{aligned} \tag{10}$$

where α_1 and α_2 are the corresponding coefficients in equation (8) and β_{IW} is the corresponding coefficient in equation (9). There are thus five expectations terms that appear in the consumption block: $\mathrm{PV}\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1}$, $\mathrm{PV}^2\Delta\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1}$, $\mathrm{PV}\left(\Delta r_{REA,10}\right)_{t|t-1}$, $\mathrm{PV}\left(\Delta \bar{i}_{EA}\right)_{t|t-1}$ and $\mathrm{PV}\left(\Delta \bar{\pi}_{EA}\right)_{t|t-1}$. We next describe the construction of these variables, the policy functions and auxiliary equations associated with each of these five terms.

Table 2.5.4 describes $\mathrm{PV}\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1}$, the expectation of the logarithm of the ratio of household income to long run output. Note that the construction of this variable is based on trend-cycle decomposition using the Hodrick-Prescott filter, which is motivated by the fact that there is a downward trend in the income-output ratio, as shown in Figure 2.5.2(b).[15] That is, in estimation we apply the filter to the logarithm of the ratio of household income to long run output to decompose it into these two components: the trend $\overline{\tau}_{REA,YH,\bar{Y},t}$ and cycle $\hat{\tau}_{REA,YH,\bar{Y},t}$. The PAC framework for constructing expectations variables is then applied to these two terms individually. The expectation term $\mathrm{PV}\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1}$ is then computed as the sum of the expectation of the cyclical

[15]This is possibly due to the declining labor share in Germany.

component PV $\left(\hat{\tau}_{REA,YH,\bar{Y}}\right)_{t|t-1}$ and the expectation of the trend component PV $\left(\overline{\tau}_{REA,YH,\bar{Y}}\right)_{t|t-1}$. The policy function is then estimated using OLS for this sum term PV $\left(y_{REA,H} - \bar{y}_{REA}\right)_{t|t-1}$.

Table 2.5.4: Coefficients of the policy function and auxiliary equation for the expectation of the logarithm of the income-long run output ratio

VAR model	**Policy function** PV $\left(\boldsymbol{y_{REA,H} - \bar{y}_{REA}}\right)_{\boldsymbol{t\|t-1}}$	**Auxiliary equation** $\boldsymbol{\hat{\tau}_{REA,YH,\bar{Y}}}$	**Auxiliary equation** $\boldsymbol{\overline{\tau}_{REA,YH,\bar{Y}}}$
$\widehat{y}_{t-1}$	-0.004		
$i_{EA,t-1} - \bar{i}_{EA,t-1}$	-0.18		
$\pi_{t-1} - \bar{\pi}_{t-1}$	-0.003		
$\widehat{y}_{EA,t-1}$	0.04		
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.01		
$\widehat{y}_{REA,t-1}$	0.005	0.03 [0.02]	
$\hat{\tau}_{REA,YH,\bar{Y},t-1}$	0.13	0.73 [0.07]	
$\overline{\tau}_{REA,YH,\bar{Y},t-1}$	1		1

Note: standard errors in brackets. $R^2 = 0.59$ for the auxiliary equation of the cyclical component

Table 2.5.5 presents the policy function for the expected present value of current and future changes of permanent income. Note that the corresponding auxiliary equation of PV $\left(y_{REA,H}\right)_{t|t-1}$ is in fact given by the policy function described in Table 2.5.4. The policy function of expected permanent income provides weights of the usual core variables of the VAR, of the cyclical/trend components of the income ratio and of the lagged permanent income. As lagged permanent income and current permanent income are both within the information set of household (past values of observed variables until $t-1$), the expectation of the current change of permanent income is equal to the current change itself. For this reason, the weight of the lagged permanent income is simply the opposite of the discount factor at the current date t (-0.16), a discount factor always equal to the error-correction coefficient of the PAC equation (equal here to 0.16, as reported in Table 2.5.3).[16] For the same reason, the lag of the trend component of the income ratio, which appears with a unit weight within current income, appears within the policy function with the opposite weight.[17]

We then turn to the construction of the expectation of the REA real 10-year sovereign bond rate. As the auxiliary equation for this variable is a complicated linear transformation of the E-SAT variables, we present the corresponding estimation results separately in Table 2.5.6. The estimated

[16]For an explanation of the equality between the discount factor at current date t within the present value variable of a PAC equation and the error-correction coefficient of this equation, see the note "Polynomial Adjustment Costs in FRB/US" of the documentation of the FRB/US model.

[17]Future changes of the trend component of the income ratio, which appear in future changes of permanent income, do not matter. Indeed, as the trend component of the income ratio is simply modeled as a random walk, the expectations of all future changes of this ratio are all equal to zero.

Table 2.5.5: Coefficients of the policy function for the expectation of permanent income

VAR model	Policy function $\mathbf{PV^2\Delta\left(y_{REA,H}-\bar{y}_{REA}\right)_{t\mid t-1}}$
$\widehat{y}_{t-1}$	-0.0002
$i_{EA,t-1}-\bar{i}_{EA,t-1}$	-0.02
$\pi_{t-1}-\bar{\pi}_{t-1}$	-0.0002
$\widehat{y}_{EA,t-1}$	0.003
$\pi_{EA,t-1}-\bar{\pi}_{EA,t-1}$	0.0003
$\widehat{y}_{REA,t-1}$	0.0002
$\hat{\tau}_{REA,YH,\bar{Y},t-1}$	0.004
$\bar{\tau}_{REA,YH,\bar{Y},t-1}-\mathrm{PV}\left(y_{REA,H}\right)_{t-1\mid t-2}$	0.09

equation is of the form

$$\begin{aligned} r_{REA,10,t} \quad = \quad & \gamma_0 r_{REA,10,t-1}+(1-\gamma_0)\left(\bar{i}_{EA,t-1}-\bar{\pi}_{EA,t-1}+\alpha\right) \\ & +\gamma_1\left(i_{EA,t-1}-\bar{i}_{EA,t-1}\right) \\ & +\gamma_2\left(\pi_{EA,t-1}-\bar{\pi}_{EA,t-1}\right)+\varepsilon_t \end{aligned} \tag{11}$$

where α is the average term premium on these REA bonds, estimated separately to be 0.002.[18] The estimation results for the γ_i are presented in Table 2.5.6, while Table 2.5.7 presents the estimation results for the policy function of $\mathrm{PV}\left(\Delta r_{REA,10}\right)_{t\mid t-1}$.

Table 2.5.6: Coefficients of the auxiliary equation for the expectation of the real 10-year sovereign bond rate for REA

Coefficient	Estimate
γ_0	0.94 [0.037]
γ_1	0.01 [0.036]
γ_2	0.004 [0.038]

Note: standard errors in brackets. $R^2=0.96$

The final expectation terms are described in Table 2.5.8 which presents the policy functions for $\mathrm{PV}\left(\bar{i_{EA}}\right)_{t\mid t-1}$ and $\mathrm{PV}\left(\bar{\pi}_{EA}\right)_{t\mid t-1}$. The auxiliary equations for these terms are simply the E-SAT core equations as described in Lemoine et al. (2019), implying that appropriately defined policy functions depend only on the two variables themselves.

Primary income of households The relationship between real disposable income $Y_{REA,H,t}$ and the household's primary income – $\tilde{Y}_{REA,H,t}$ – warrants some additional discussion. As STREAM is on purpose built to be a smaller scale model than e.g. FR-BDF, ECB-BASE and FRB/US, some features of the Euro area economy have been modeled using stronger simplifications than in these

[18]See Section 2.9 for details.

Table 2.5.7: Coefficients of the policy function for the expectation of the real 10-year sovereign bond rate for REA

VAR model	Policy function $\mathbf{PV}\left(\Delta r_{REA,10}\right)_{t\mid t-1}$
Intercept	$4.14 \cdot 10^{-5}$
$\widehat{y}_{t-1}$	$6.42 \cdot 10^{-6}$
$i_{EA,t-1}$	0.0023
$\bar{i}_{EA,t-1}$	0.0249
π_{t-1}	$1.95 \cdot 10^{-5}$
$\bar{\pi}_{t-1}$	$5.19 \cdot 10^{-5}$
$\widehat{y}_{EA,t-1}$	0.0001
$\pi_{EA,t-1}$	0.0003
$\bar{\pi}_{EA,t-1}$	-0.0212
$\widehat{y}_{REA,t-1}$	0
$\pi_{REA,t-1}$	$-6.31 \cdot 10^{-5}$
$\hat{\tau}_{REA,YH,\bar{Y},t-1}$	0
$\overline{\tau}_{REA,YH,\bar{Y},t-1}$	0
$\text{PV}\left(y_{REA,H} - \bar{y}_{REA}\right)_{t\mid t-1}$	0
$r_{REA,10,t-1}$	-0.03

Table 2.5.8: Coefficients of the policy functions of expectations of $\bar{i}_t$ and $\bar{\pi}_{EA,t}$

VAR model	Policy function $\mathbf{PV}\left(\Delta \bar{i_{EA}}\right)_{t\mid t-1}$	Policy function $\mathbf{PV}\left(\Delta \bar{\pi}_{EA}\right)_{t\mid t-1}$
$\bar{i}_{EA,t-1} - \bar{i_{EA}}$	-0.009	-
$\bar{\pi}_{EA,t-1} - \bar{\pi}_{EA}$	-	-0.030

Note: Auxiliary equations defined in E-SAT core equations other similar models. Household income is one of them: instead of modeling explicitly the various income flows that disposable income is in reality (and in these other models) comprised of, we assume that the intertemporal dynamics of this quantity can be modeled adequately using an error correction equation relating primary income to nominal output. Note that we measure primary income as the sum of nominal disposable income and taxes paid by the household less transfers received.

More specifically, we assume that

$$Y_{REA,H,t} = (1 - \tau_{REA,D5,YH,t}) \left(\tilde{Y}_{REA,H,t} + T_t\right) / P_{REA,C,t} \tag{12}$$

$$\begin{aligned} \Delta \log \tilde{Y}_{REA,H,t} \quad = \quad & (1 - \beta_1 - \beta_3 - \beta_4) (\bar{g} + \bar{\pi}) \\ & + \beta_1 \Delta \log \tilde{Y}^*_{REA,H,t} \\ & + \beta_2 \log \left(\tilde{Y}_{REA,H,t-1} - \tilde{Y}^*_{REA,H,t-1}\right) \\ & + \beta_3 \Delta \log \tilde{Y}_{REA,H,t-1} + \beta_4 \Delta \log \tilde{Y}_{REA,H,t-2} + \varepsilon_t \end{aligned} \tag{13}$$

and

$$\log \tilde{Y}^*_{REA,H,t} = \gamma_0 + \log \left(Y_{REA,t} P_{Y_{REA},t}\right) + \gamma_1 t \tag{14}$$

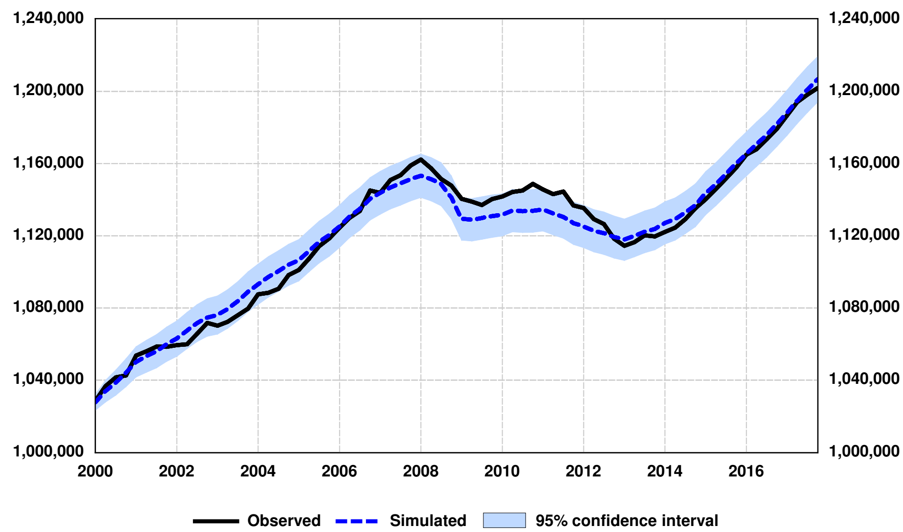
where the first equation states that real disposable income $Y_{REA,H,t}$ is determined as the time-varying share $1 - \tau_{REA,D5,YH,t}$ of the sum of primary income $\tilde{Y}_{REA,H,t}$ and transfers T_t, where $\tau_{REA,D5,YH,t}$ is the ratio of taxes paid by the household to their income; in the long run it converges to 0.14.

Equation (13) states that in simulation primary income is assumed to follow an error correction model. The final component of this block, equation (14), describes the determination of the target – $\log \tilde{Y}^*_{REA,H,t}$ – used in the error correction equation as a affine transformation of the logarithm of nominal output and a time trend. The trend is due the previously discussed decline in the ratio of income to output, as demonstrated in Figure 2.5.2(b). Note that we relate nominal primary income to nominal output instead of having a relationship for their volumes in order to capture a term-of-trade effect in a reduced-form way.

Dynamic simulation Figures 2.5.2 and 2.5.3 show observed and simulated household consumption and income for REA using equations (8) for simulated consumption and equations (13) and (14) for income.

The model performs for the most part rather well in matching the actual dynamics of consumption. It does a particularly good job in the the period leading up to the Great Financial Crisis, i.e. up to 2007, and in the post-Crisis recovery starting around 2013. In the very last periods before the onset of the crisis at the end of 2007 the model fails in fully capturing the growth of consumption as it does during the 2010 and 2011, but these misses are relatively minor compared to the overall dynamics during this difficult period.

Figure 2.5.2: Household consumption, in volume



Notes: confidence interval for the dynamic simulation is obtained by stochastic simulations from bootstrapped residuals (sample 2000Q1-2017Q4).

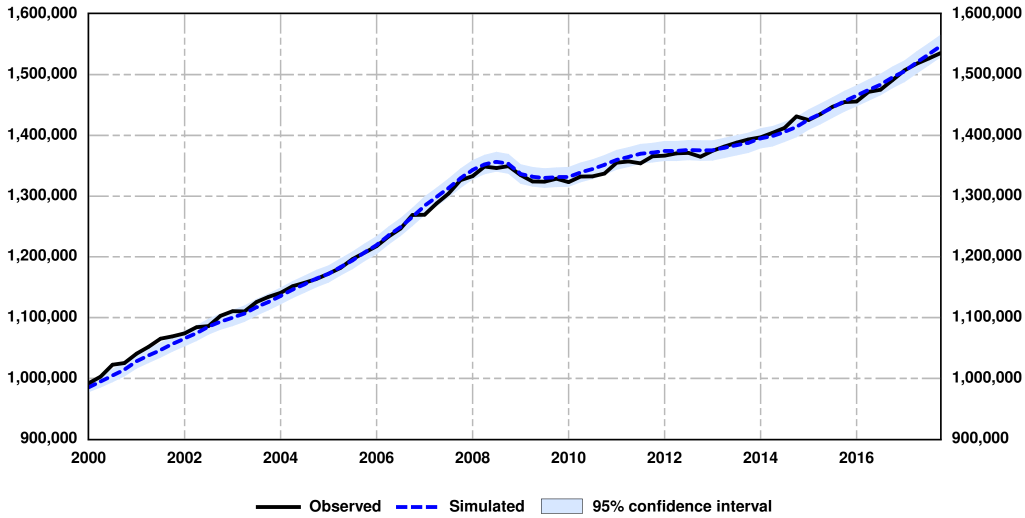
For simulated income the performance is even better. As can be seen from Figure 2.5.3, the match between observed and simulated income is particularly tight in the years before the crisis and in the years of the immediate recovery. A somewhat weaker, if still good fit can be observed during the crisis years, where the simulation predicts a higher income level than is actually observed, and at the very start of the sample – 2000 to 2003 – where observed income is persistently higher than simulated, and also outside the 95% confidence interval.

2.6 Total investment

In STREAM, we model aggregate total investment, including public investment, within a single PAC equation, in order to keep the model as small and parsimonious as possible.[19] Our equations are a simplified version on the business investment block of FR-BDF, which is derived from a first order condition for capital demand for a firm with a CES production function. The long-run equation defines the target of total investment, which depends on aggregate demand and an approximated measure of the expected real user cost capital. The short-run equation follows the standard PAC structure, with roles for short-run deviation from target investment, expectations of target growth and current aggregate demand (excluding total investment).

[19] We also estimated an equation for aggregate *private* investment and found similar estimates compared to aggregate total investment. These findings make us confident that using a single block for total investment does not impair our model properties compared to an alternative in which we would have modeled them separately.

Figure 2.5.3: Household income, in volume



Notes: confidence interval for the dynamic simulation is obtained by stochastic simulations from bootstrapped residuals (sample 2000Q1-2017Q4).

Target The target of aggregate investment $I^*_{REA,t}$ is defined by eq. (15) and estimates are reported in Table 2.6.2. It is an approximation of FR-BDF business investment demand equation, which is itself derived from a standard profit maximization problem for a firm that has a CES production function and no investment adjustment costs. We depart from the theoretical framework in three dimensions. First, we omit the log of steady-state investment-to-capital ratio, since we do not model the capital stock for REA, and its role is implicitly taken into account within the intercept of the equation. Second, as we model here total investment, it depends here on total GDP, $y_{REA,t}$, in the long-run and not on the value-added of market branches. Third, we approximate the relationship between the log of the real user cost of capital $r_{K,REA,t}$ by linearizing it around a measure of the "neutral" real cost of capital $\overline{r}_{K,REA,t-1}$. We describe in more details $r_{K,REA,t}$ and $\overline{r}_{K,REA,t}$ further below.

Our choice to linearize the log of the real user cost of capital has several motivations. First, an investment demand equation with the *log* of real user cost of capital yields inconsistent estimates for σ, due to a downward trend in the real user cost of capital in the estimation sample.[20] The relative deviation of the real cost of capital from its neutral level is more stable, thanks to the downward trend of the neutral rate (see Figure 2.6.1). Second, our real user cost of capital depends directly on the 10yr sovereign bond rate, plus a small depreciation rate and minus expected inflation. Hence, given the omission of risk premia of various components of the WACC (bond rate, bank lending rate and cost of equity), if expected inflation were too large, it could be possible that the real user cost

[20]The measure of real cost of capital in FR-BDF is less trended, since it depends on a measure of WACC that includes the cost of equity.

Table 2.6.1: Variables used in section 2.6

Notation	Description
$I_{REA,t}$	Total investment in REA, volume
$I^*_{REA,t-1}$	Target of total investment in REA, volume
$r_{K,REA,t}$	Real user cost of capital in REA
$\overline{r}_{K,REA,t-1}$	Neutral real user cost of capital in REA
$y_{REA,t}$	GDP in REA, volume, in log
$i_{REA,10,t}$	10-yr sovereign bond rate in REA
$\text{PV}(\pi_{REA})_{t\|t-1}$	Expected present value of inflation in REA (GDP deflator price)
$\log \widehat{I}^*_{REA,t}$	Log-deviation of target investment from its trend in REA
$\delta_{09q1,t}$	Dummy variable equal to 1 during 2009Q1
E-SAT	See Table 2.2.1

becomes negative, which further motivates the choice of a linearized equation, to avoid using logs. As a result, the elasticity of the target $I^*_{REA,t}$ to the real user cost of capital $r_{K,REA,t}$ is represented by the parameter σ that is estimated to be 0.24. This value for sigma is lower than the one in FR-BDF (0.53) but cannot be compared to it since both we FR-BDF's elasticity of substitution σ was estimated for business investment and with a different definition of the real user cost of capital.

$$\log I^*_{REA,t} = \alpha_0 + y_{REA,t} - \sigma \frac{r_{K,REA,t} - \overline{r}_{K,REA,t}}{\overline{r}_{K,REA,t}} \tag{15}$$

Table 2.6.2: Coefficients and standard errors of the long-run equation for total investment

Coefficient	Estimate	s.e.
α_0	-1.53	$0.79 \cdot 10^{-2}$
σ	0.26	$2.59 \cdot 10^{-2}$

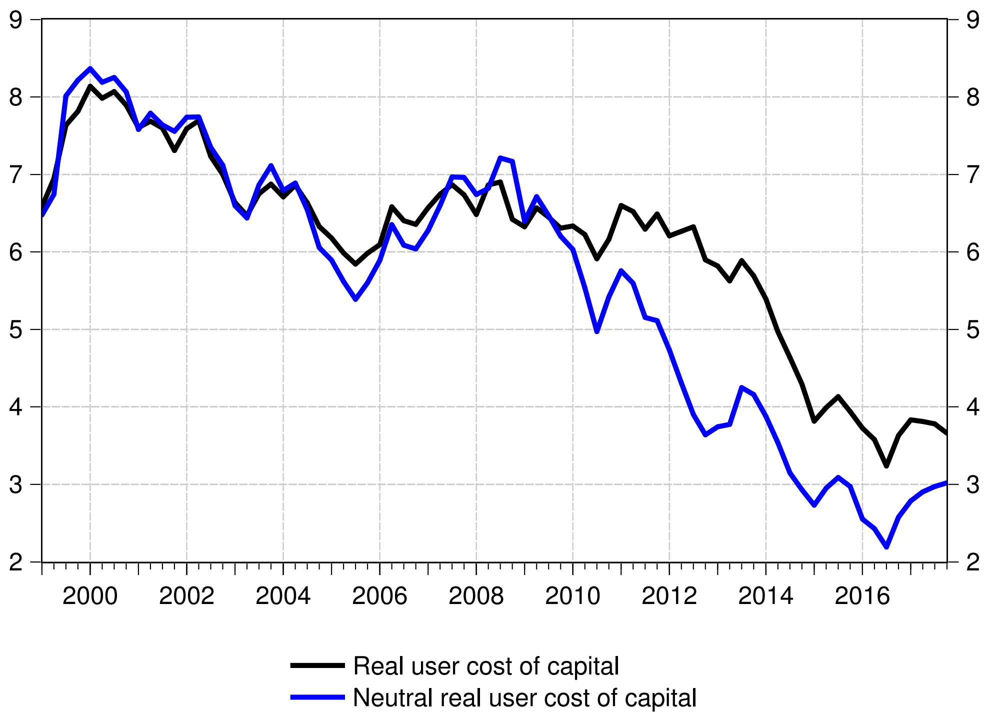
$R^2 = 0.76$

Real user cost of capital In STREAM, the real user cost of capital depends on the 10yr-sovereign bond rate and expected inflation, such that:

$$r_{K,REA,t} = i_{REA,10,t} + \delta_{K,REA} - \text{PV}(\pi_{REA})_{t|t} \tag{16}$$

where δ is a constant depreciation rate for aggregate capital stock (including housing capital). Since this parameter is fairly stable over time, we calibrate it to 4.6% annually based on previous estimates for the euro area aggregate capital stock (ECB, 2006). In contrast, in FR-BDF, the user cost of capital is based on a Weighted Average Cost of Capital (WACC), which components (cost of equity,

Figure 2.6.1: Real user cost of capital and its neutral rate, in annualized pp



bank lending rates and bond rates for firms) are modeled as spreads over the 10-yr sovereign french bond rate. As a result, the real user cost of capital in STREAM is *lower* than its counterpart in FR-BDF, as it directly depends on the REA sovereign bond rate $i_{REA,10,t}$.[21] The neutral real user cost of capital is defined from the long-run anchor of short-term interest rate $\bar{i}_{EA,t-1}$ and inflation $\bar{\pi}_{EA,t-1}$ for the euro area (measured by 5-year futures of the 3-month Euribor rate and the long-run professional consensus forecast), the depreciation rate δ and the steady-state term premium of the 10yr bond rate over the short-rate $\bar{s}_{REA,10}$ such that:

$$\bar{r}_{K,REA,t} = \bar{i}_{EA,t} + \bar{s}_{REA,10} + \delta_{K,REA} - \bar{\pi}_{EA,t} \tag{17}$$

Finally, expected present-value of inflation for REA is constructed using E-SAT, extended with REA output gap and inflation; the policy function is detailed in Table 2.6.3. In this case, expected inflation is computed by using the extended version of E-SAT but can nonetheless be expressed using only a linear combination of France and euro area variables, since REA output gap and inflation rate are defined themselves as such linear combinations.

[21] The 10yr sovereign bond rate for REA is constructed as a GDP-weighted average of the Big-4 sovereign bond rates; see section 2.9 for details.

Table 2.6.3: Coefficients of the policy function for expected inflation

VAR model	Coefficient PV $(\pi_{Y,REA})_{t\|t-1}$
Constant	$0.27 \cdot 10^{-2}$
$\widehat{Y}_{t-1}$	$-0.88 \cdot 10^{-2}$
π_{t-1}	$-2.30 \cdot 10^{-2}$
$i_{EA,t-1} - \bar{i}_{EA,t-1}$	$-7.42 \cdot 10^{-2}$
$\widehat{y}_{EA,t-1}$	$2.33 \cdot 10^{-2}$
$\pi_{EA,t-1}$	$4.42 \cdot 10^{-2}$
$\bar{\pi}_{t-1}$	$-9.46 \cdot 10^{-2}$
$\bar{\pi}_{EA,t-1}$	$50.3 \cdot 10^{-2}$

Short run equation The short run dynamics of investment are determined by a second order PAC equation (18). The estimated coefficients are presented in Table 2.6.4. Regarding expectations, we depart from the standard PAC structure and only include the stationary component $\text{PV}\left(\Delta \log \widehat{I}^*_{REA}\right)_{t|t-1}$ of expectations, which is the expected present-value of investment target growth rate (in deviation from its trend).[22] The additional *ad hoc* term, $\Delta \log(Y_{REA,t} - I_{REA,t})$, is the (log) growth rate of real GDP excluding total investment that accounts for an accelerator effect of aggregate demand on total investment and improves the fit of the equation.[23] Finally, we include a dummy variable equal to 1 in 2009Q1 to capture the sharp drop of investment during the Great Recession.

$$\begin{aligned}\Delta \log I_{REA,t} \quad = \quad & \beta_0 \log\left(\frac{I^*_{REA,t-1}}{I_{REA,t-1}}\right) + \beta_1 \Delta \log I_{REA,t-1} + \beta_2 \Delta \log I_{REA,t-2} \\ & + \text{PV}\left(\Delta \log \widehat{I}^*_{REA}\right)_{t|t-1} \\ & + (1 - \beta_1 - \beta_2 - \beta_3)\, \Delta \bar{y}_{REA,t-1} \\ & + \beta_3 \Delta \log(Y_{REA,t} - I_{REA,t}) \\ & + \beta_4 \delta_{09q1,t} + \varepsilon_t \end{aligned} \tag{18}$$

Expectations Expected present-value of target investment's growth rate gap is constructed using E-SAT and the following AR(1) auxiliary equation:

$$\log \widehat{I}^*_{REA,t} = \beta_1 \log \widehat{I}^*_{REA,t-1} + \beta_2 \widehat{y}_{REA,t-1} + \varepsilon_t \tag{19}$$

that relates the gap between investment target and its trend (in log) to its own lag and the lag of REA output gap. Estimates of equation (19) are reported in Table 2.6.5, along with the policy function

[22] This choice is, in fact, without consequences since the non-stationary (trend) component of expectations would cancel out with the growth neutrality term.

[23] We exclude total investment as it would create an obvious endogeneity bias, as GDP includes investment.

Table 2.6.4: Coefficients and standard errors of the short-run equation for total investment

Coefficient	Estimate	s.e.
β_0	0.07	0.04
β_1	0.17	0.09
β_2	0.33	0.09
β_3	0.66	0.27
β_4	-0.04	0.01

$R^2 = 0.55$

for the expectation term.[24] As usual in the PAC framework, the negative coefficient associated to lagged investment target gap (-0.046) is interpreted as a dampening effect: following a transitory shock to the target of investment, in the VAR-based case, agents expect target to revert toward its trend, which "dampens" the increase in investment demand.

Table 2.6.5: Coefficients of the policy function and auxiliary equation for the expectation of investment target growth gap

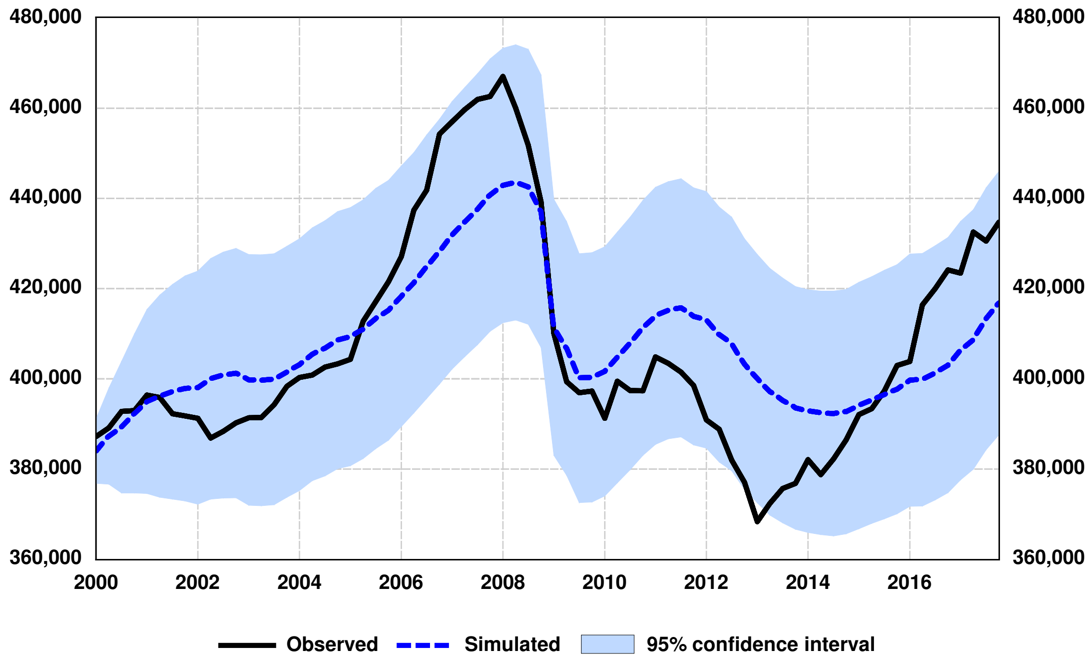
VAR model	Policy function $\mathbf{PV}\left(\Delta \log \widehat{I}^*_{REA}\right)_{t\|t-1}$	Auxiliary equation
$\widehat{y}_{t-1}$	-0.18 $\cdot 10^{-2}$	
$i_{EA,t-1} - \bar{i}_{EA,t-1}$	-4.14 $\cdot 10^{-2}$	
$\pi_{t-1} - \bar{\pi}_{t-1}$	-0.13 $\cdot 10^{-2}$	
$\widehat{y}_{EA,t-1}$	1.73 $\cdot 10^{-2}$	
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	1.02 $\cdot 10^{-2}$	
$\widehat{y}_{REA,t-1}$	0.26 $\cdot 10^{-2}$	0.10 [0.07]
$\log \widehat{I}^*_{REA,t-1}$	-5.20 $\cdot 10^{-2}$	0.77 [0.09]

Note: standard errors in brackets. $R^2 = 0.68$ for the auxiliary equation

Dynamic simulation Figure 2.6.2 presents observed and simulated total investment for REA using equation (15). Despite the simplifying assumptions we made, our equation shows fairly good properties in dynamic simulations. We are able to reproduce the double-dip recession following the sovereign debt crisis, although the equation cannot fully capture the boom in 2006-2008 and the burst in 2011-2013. Though it is almost likely impossible to get a satisfying empirical fit for *aggregate* REA investment, since both the boom and the burst are specific to some countries in the REA (e.g. Spain) while mostly absent in others (e.g. Germany).

[24] Because the investment target gap specifically depends on the REA output gap, this last variable appears in the expectation policy function, which cannot be completely expressed using a linear combination of France and euro area output gaps.

Figure 2.6.2: Total investment, in volume



Notes: confidence interval for the dynamic simulation is obtained by stochastic simulations from bootstrapped residuals (sample 2000Q1-2017Q4).

2.7 External trade

The external trade block is formed of four equations modeling internal and consolidated imports and exports. Internal imports and exports are those among countries of the REA, whereas consolidated imports and exports are those between the REA and both France and the rest of the world. Defined this way, internal and consolidated trade flows are the equivalent at the REA level of the intra/extra decomposition at the Euro Area level. See appendix A.1 for a detailled discussion of how we constructed these time series.

Internal Exports The real internal exports equation is estimated as real internal imports plus an AR(1). The estimation period is 2003Q1-2017Q4. It reflects the fact that the internal trade should be balanced: inside the REA, aggregate exports to REA countries should be equal to aggregate imports from REA countries, with a minor discrepancy due to measurement errors and accounting techniques[25].

[25] Exports are generally measured franco on board (FOB) whereas imports are measured including insurance and freight costs (CIF).

Table 2.7.1: Variables used in the section 2.7

Notation	Description
$X_{REA,INT,t}$	REA internal exports, volume
$X_{REA,CON,t}$	REA consolidated exports, volume
$X_{REA,t}$	REA total exports, volume
$M_{REA,INT,t}$	REA internal imports, volume
$M_{REA,CON,t}$	REA consolidated imports, volume
$P_{REA,X,CON,t}$	REA consolidated export price
$P_{REA,M,CON,t}$	REA consolidated import price
$P_{REA,CX,t}$	REA foreign competitors' price (export side)
Ω_t	Weight of emerging countries
$\bar{Y}_{REA,t}$	Long-run trend of REA GDP, volume
$WD_{REA,t}$	World demand for REA, volume
$WS_{REA,t}$	World supply for REA, volume
$IAD_{REA,t}$	Import intensity-adjusted measure of aggregate demand (IAD) for the REA
$D_{REA,t}$	REA real demand proxied by GDP plus real total imports
$\Delta\bar{q}$	Long-run anchor of the output growth rate
$C_{REA,t}$	REA household consumption, volume
$C_{REA,G,t}$	REA government consumption, volume
$I_{REA,t}$	REA total investment, volume

$$x_{REA,INT,t} = m_{REA,INT,t} + u_t \quad (20)$$

$$u_t = \rho_0 u_{t-1} + \epsilon_t \quad (21)$$

For notations, see Table 2.7.1. Estimated coefficients are shown in Table 2.7.2.

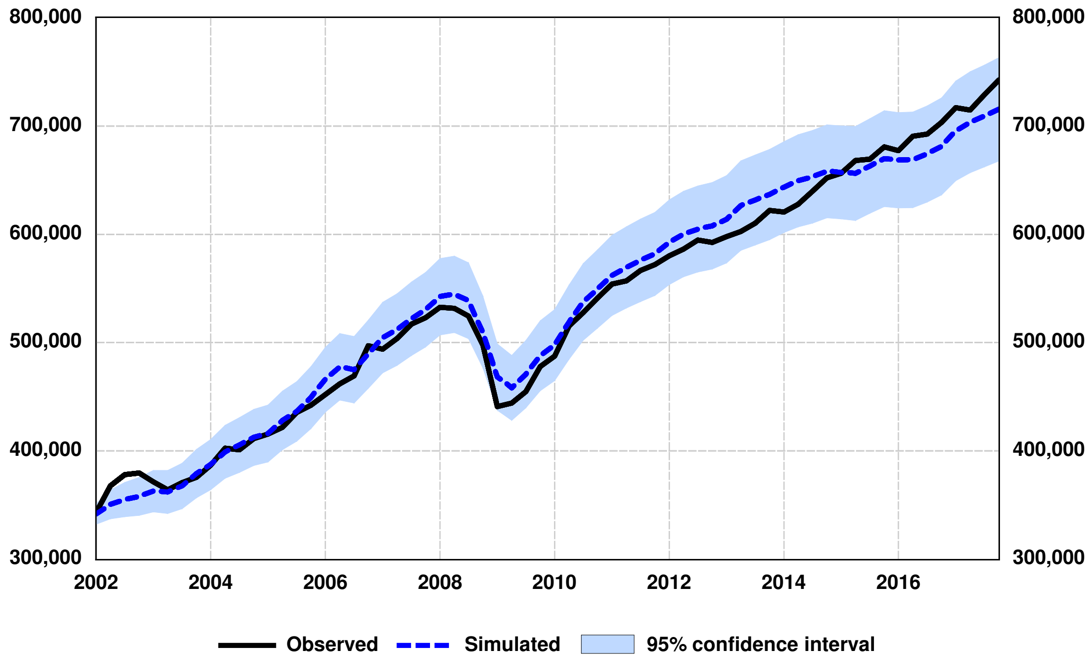
Table 2.7.2: Coefficients and standard errors of the internal exports volume

Coefficient	Estimate	s.e.
ρ_0	0.98	0.02

$R^2 = 0.32$

Consolidated Exports The real consolidated exports equation is estimated as a one-step ECM, with a very similar specification to what one can find in the FR-BDF model Lemoine et al. (2019). The estimation period is 2002Q1-2017Q4.

Figure 2.7.1: REA consolidated exports, volume



Target In the long-run, consolidated exports follow world demand adressed to the REA[26] with a unit elasticity to account for the fact that in the long-run supply and demand are balanced. The ratio $\frac{P_{REA,X,CON,t}}{P_{REA,CX,t}}$ is a price competitiveness indicator of REA consolidated exports. Finally, the weight of emerging countries in world trade is a proxy for non-price competitiveness factors, which are not properly taken into account by the price competitiveness variable since it only captures market shares at the extensive margin rather than at the intensive margin.

$$x^*_{REA,CON,t} = \beta_0 + wd_{REA,t} + \beta_1(p_{REA,X,CON,t} - p_{REA,CX,t}) + \beta_2 \log(\Omega_t) \tag{22}$$

For notations, see Table 2.7.1. Estimated coefficients are shown in Table 2.7.3.

Short run equation The short-run dynamic of REA real consolidated exports is only explained by world demand for the REA in volume and the error-correction term. Notice that the elasticity to world demand, which was not significantly different from unity, is constrained to be equal to one.

$$\Delta x_{REA,CON,t} = \Delta wd_{REA,t} + \beta_0 \left[x_{REA,CON,t-1} - x^*_{REA,CON,t-1}\right] + \epsilon_t \tag{23}$$

[26] This variable is calculated as a weighted sum of imports by trade partners of the REA (which includes France). The formula is in the spirit of those used in Eurosystem forecat exercices. See Hubrich & Karlsson (2010) for more details about the general formula and appendix A.1 for details about how these are transposed for the REA.

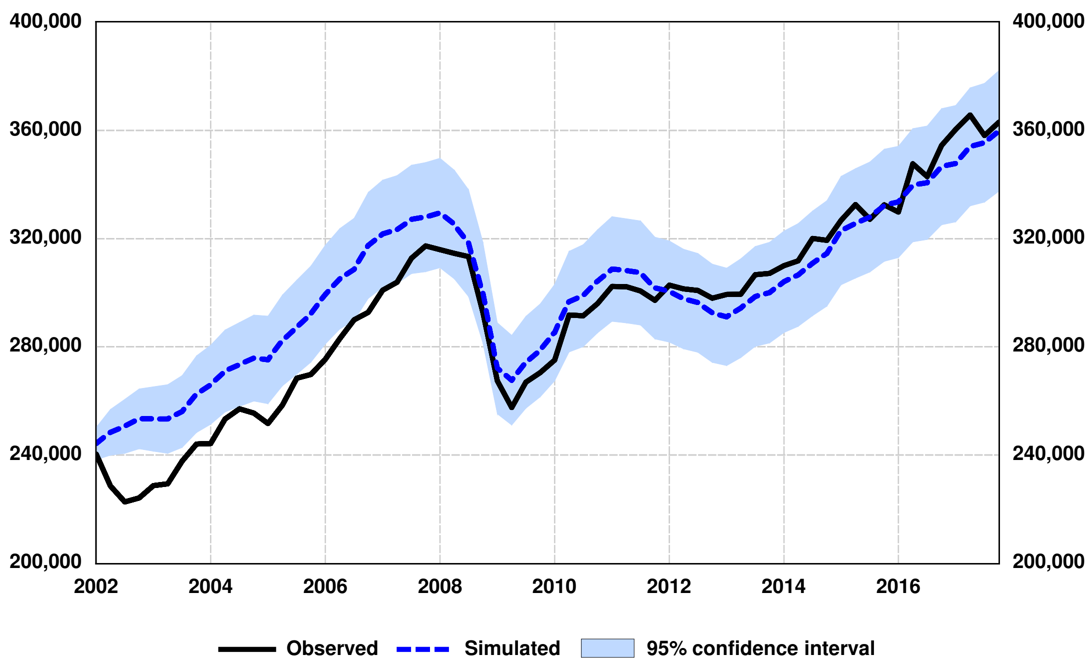
Table 2.7.3: Coefficients and standard errors of the consolidated exports volume

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	13.24	0.30	-0.09	0.05
β_1	-0.90	0.88		
β_2	-0.07	0.27		

$R^2 = 0.70$ (1 step estimation)

Internal imports The real internal imports equation is estimated as a one-step ECM. The estimation period is 2003Q1-2017Q4.

Figure 2.7.2: REA internal imports, volume



Target Internal REA imports are pegged to the REA total demand (the REA real GDP plus real total imports) with a unit elasticity.

$$m^*_{REA,INT,t} = \beta_0 + d_{REA,t} \tag{24}$$

Table 2.7.4: Coefficients and standard errors of the internal imports volume

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	-2.23	0.01	2.01	0.10
β_1			-0.08	0.02

$R^2 = 0.74$ (1 step estimation)

Short run equation

$$\Delta m_{REA,INT,t} = \beta_0 \Delta d_{REA,t} + (1-\beta_0)\Delta\bar{q} + \beta_1 \left[m_{REA,INT,t-1} - m^*_{REA,INT,t-1}\right] + \epsilon_t \tag{25}$$

Notice that the estimated coefficient on REA total demand is larger than 1, which means that, in the short run, additional demand is served by a larger and more diversified foreign supply.

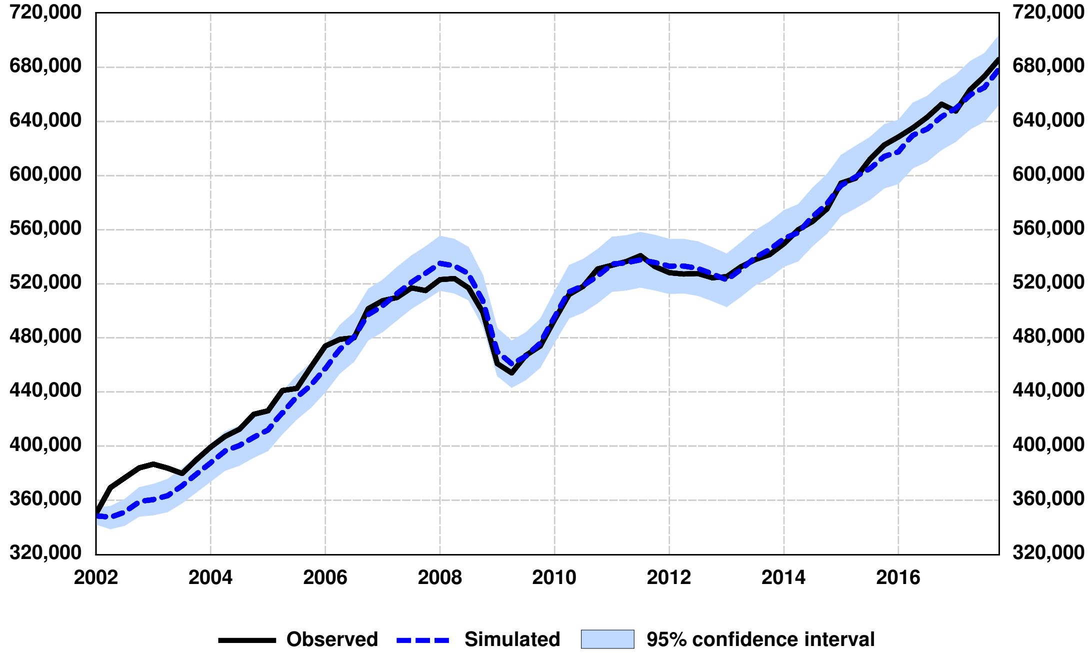
Consolidated imports The real consolidated imports equation is estimated as a one-step ECM. The estimation period is 2003Q1-2017Q4.

Consolidated imports closelly follow an import intensity-adjusted measure of aggregate demande (IAD) for the REA. This variable reflects the import content in each component of the REA demand. It is calculated as a weighted geometric mean of demand side components (private and public consumptions, investment and total exports) using mean import contents among REA countries from Bussière et al. (2020) data (see table 1.a of the paper). Equation (26) shows the exact formula used to compute this variable.

$$iad_{REA,t} = 0.252c_{REA,t} + 0.098c_{REA,G,t} + 0.332i_{REA,t} + 0.321x_{REA,t} \tag{26}$$

Target The target equation used for consolidated imports is likewise very similar to the one used for French imports in FRBDF (see Lemoine et al. (2019)). In the long-run, consolidated imports are pegged to the REA demand (calculated as REA real GDP plus real total REA imports). The ratio $\frac{P_{REA,Mxn,CON,t}}{P_{REA,X,CON,t}}$ represents the REA price-competitiveness. We use the non-energy import price to avoid price-competitiveness effects in case of a shock to oil price, following Angelini et al. (2019). We used the export price instead of the value added price because the first one is a better proxy of tradable goods prices produced locally than the second one. The implicit assumption here is that domestic firms charge the same price domestically and on foreign markets for a similar bundle of goods and services sold on both markets (whereas value added prices include a large share of non-traded goods and services). Finally, the ratio $\frac{\bar{Y}_{REA,t}}{WS_{REA,t}}$ is a measure of the relative variety of goods in the REA relative to the world (including France). The idea is that foreign firms can capture market share in the REA not only by charging lower prices but also by adding new varieties. The value added trend $\bar{Y}_{REA,t}$ and the world supply $WS_{REA,t}$ are used as proxies for varieties respectively in

Figure 2.7.3: REA consolidated imports, volume



the REA and in the rest of the world.

$$m^*_{REA,CON,t} = \beta_0 + d_{REA,t} + \beta_1(p_{REA,Mxn,CON,t} - p_{REA,X,CON,t}) + \beta_2(\bar{y}_{REA,t} - ws_{REA,t}) \quad (27)$$

Table 2.7.5: Coefficients and standard errors of the consolidated import volume

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	6.10	0.87	1.37	0.07
β_1	-1.49	0.65	-0.12	0.05
β_2	-0.53	0.06	0.03	0.003

$R^2 = 0.78$ (1 step estimation)

Short run equation

$$\Delta m_{REA,CON,t} = \beta_0 \Delta iad_{REA,t} + (1-\beta_0)\Delta\bar{q} + \beta_1\left[m_{REA,CON,t-1} - m^*_{REA,CON,t-1}\right] + \beta_2\delta_{2010q1} + \epsilon_t \quad (28)$$

In the short-run equation, we capture the effect of internal demand through the weighted indicator $IAD_{REA,t}$ instead of the non-weighted indicator $D_{REA,t}$, because this alternative indicator helps improving the fit of the equation. As for the internal imports short-run equation, the estimated coefficient of internal demand is larger than 1.

2.8 Deflators

Table 2.8.1: Variables used in the section 2.8

Notation	Description
$P_{REA,X,CON,t}$	REA consolidated export price
$P_{REA,X,INT,t}$	REA internal export price
$P_{REA,M,CON,t}$	REA consolidated import price
$P_{REA,Mxn,CON,t}$	REA consolidated non-energy import price
$P_{REA,Mn,CON,t}$	REA consolidated energy import price, proxied by oil price in euros
$P_{REA,M,INT,t}$	REA internal import price
$P_{REA,CX,t}$	REA foreign competitors' price (export side)
$P_{REA,CM,t}$	REA foreign competitors' price (import side)
$P_{oil,t}$	Oil price (Brent in Dollar)
Ω_t	Weight of emerging countries
USD_t	Dollar/Euro exchange rate
$\bar{P}_{REA,\bar{Y},t}$	Price of long-run trend of REA GDP
$P_{REA,D,t}$	REA deflator of demand proxied by GDP plus total imports
$P_{REA,t}$	REA GDP deflator
$\bar{\pi}_{Q,t}$	Long-run trend of the value added price inflation
$P_{REA,C,t}$	REA consumption price deflator
T_t	Time-varying trend

Internal export deflator The internal export deflator equation is estimated as the internal import deflator plus an AR(1). The estimation period is 2003Q1-2017Q4. As for internal export volumes, it reflects the fact that the internal trade should be balanced with at most a negligible discrepancy (see paragraph 2.7).

$$\begin{aligned} p_{REA,X,INT,t} &= p_{REA,M,INT,t} + u_t && (29) \\ u_t &= \rho_0 u_{t-1} + \epsilon_t && (30) \end{aligned}$$

For notations, see Table 2.8.1. Estimated coefficients are shown in Table 2.8.2.

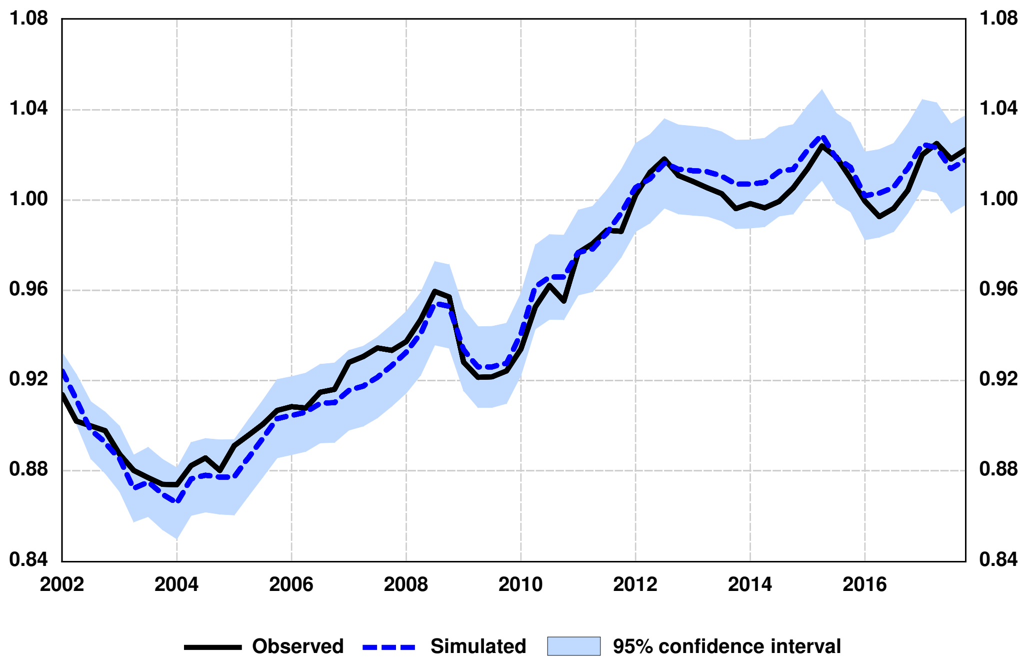
Consolidated export deflator The consolidated export deflator equation is estimated as a one-step ECM. The estimation period is 2003Q1-2017Q4.

Table 2.8.2: Coefficients and standard errors of the internal export deflator

Coefficient	Estimate	s.e.
ρ_0	0.75	0.09

$R^2 = 0.32$

Figure 2.8.1: REA consolidated export deflator



Target Consolidated REA export price in the long-run is a bundle of imports price, to account for price transmission along international value chains, domestic prices and competitor prices on the export side, to reflect incomplete pass-through (domestic firms take into account prices of foreign competitors when setting export prices). The sum of the coefficients of these variables is constrained to unity in order to have $p_{REA,X,CON,t}$ be an homogenous function of $p_{REA,M,CON,t}$, $p_{REA,t}$ and $p_{REA,CX,t}$.

$$p^*_{REA,X,CON,t} = \beta_0 + \beta_1 p_{REA,M,CON,t} + \beta_2 p_{REA,t} + (1 - \beta_1 - \beta_2) p_{REA,CX,t} \tag{31}$$

Short-run equation

$$\begin{aligned} \Delta p_{REA,X,CON,t} &= \beta_0 \Delta p_{REA,M,CON,t} + \beta_1 \Delta p_{REA,CX,t} + (1 - \beta_0 - \beta_1) \Delta p_{REA,t} \\ &\quad + \beta_2 \left[p_{REA,X,CON,t-1} - p^*_{REA,X,CON,t-1} \right] + \epsilon_t \end{aligned} \tag{32}$$

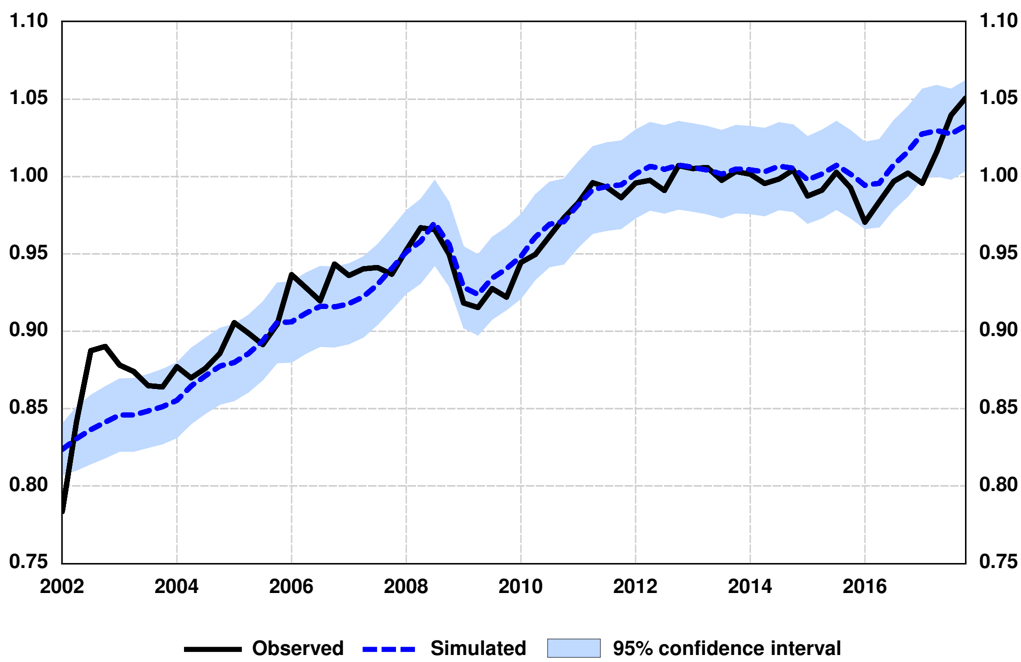
Table 2.8.3: Estimates and calibrated parameters of the consolidated export deflator

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	-0.00	0.00	0.09	0.03
β_1	0.31	-	0.24	0.03
β_2	0.55	0.08	-0.11	0.02

$R^2 = 0.74$ (1 step estimation)

Internal import deflator The internal import deflator equation is estimated as a one-step ECM. The estimation period is 2003Q1-2017Q4.

Figure 2.8.2: REA internal import deflator



Target REA internal import prices are tied in the long-run to REA domestic prices (*ie* REA GDP plus imports deflator).

$$p^*_{REA,M,INT,t} = \beta_0 + \beta_1 p_{REA,D,t} \tag{33}$$

Table 2.8.4: Estimates and calibrated parameters of the internal import deflator

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.008	0.003	1.17	0.35
β_1	1	-	0.45e-3	0.12e-3
β_2			-0.29	0.03

$R^2 = 0.38$ (1 step estimation)

Short-run equation

$$\begin{aligned} \Delta p_{REA,M,INT,t} \quad = & \quad \beta_0 \Delta p_{REA,D,t} + (1-\beta_0)\bar{\pi}_{Q,t} + \beta_1 \Delta\left(\frac{P_{oil,t-1}}{USD_{t-1}\bar{p}_{REA,\bar{Y},t-1}}\right) \\ & + \beta_2 \left[p_{REA,M,INT,t-1} - p^*_{REA,M,INT,t-1}\right] + \epsilon_t \end{aligned} \tag{34}$$

The estimated coefficient of the REA real demand is higher than one, meaning that REA internal import prices slightly overshoot in the short-run to demand shocks.

Consolidated import deflator First, the consolidated import deflator is decomposed between energy and non-energy components, in logs:

$$p_{REA,M,CON,t} = (1-\alpha_{M/Mn})p_{REA,Mxn,CON,t} + \alpha_{M/Mn}p_{REA,Mn,CON,t} \tag{35}$$

where we approximate the energy component using the oil price in euros, indexed to one at basis year 2014: $p_{REA,Mn,CON,t} = p_{oil,t} - usd_t - \ln \theta_{oil,euro}$ where $\theta_{oil,euro}$ is the oil price in euros in 2014, and $\alpha_{M/MN}$ is the long-run elasticity of consolidated total imports deflator to oil price, which is calibrated at 0.095 following Angelini et al. (2019) and cross-checked by OLS estimates.[27]

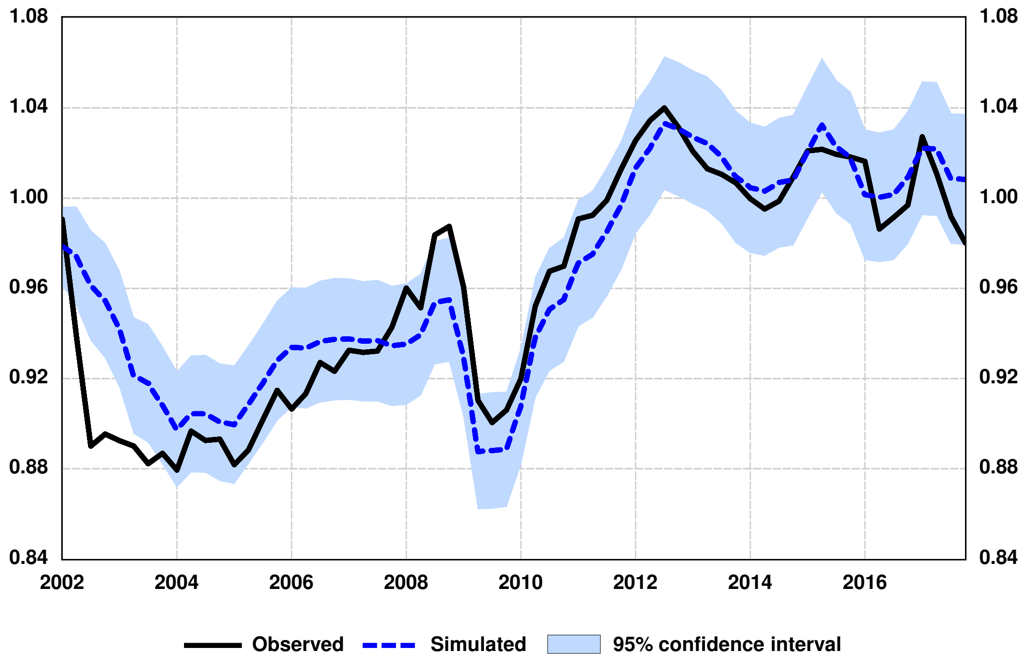
Consolidated non-energy import deflator The non-energy consolidated import deflator is estimated as a one-step ECM. The estimation period is 2003Q1-2017Q4.

Target REA consolidated non-energy import price target (36) depends on a bundle of the price of foreign exporters and the REA GDP deflator, to account for the competition between imported and locally produced goods and foreign suppliers prices in domestic currency. The share of emerging countries in world trade, Ω_t, is a proxy for the dampening effect on world prices due to the entry of new competitors from emerging markets.

$$p^*_{REA,Mxn,CON,t} = \beta_0 + \beta_1 p_{REA,CM,t} + (1-\beta_1)p_{REA,t} + \beta_2 \Omega_t \tag{36}$$

[27]This modelling choice is inspired by the ECB-BASE treatment of imports volume and deflators, see (Angelini et al., 2019). In practice, we invert equation (35) to recover the approximated non-energy consolidated import deflator in our historical sample.

Figure 2.8.3: REA consolidated non-energy import deflator



Short-run equation Short-run equation of consolidated non-energy imports deflator (37) simply depends on the growth rate of foreign exporters and a growth-neutrality constraint, in addition to the error-correction term. A dummy variable equal to one in 2009-Q2 helps to capture the sharp drop in non-energy imports price during the Great Recession.

$$\begin{aligned}\Delta p_{REA,Mxn,CON,t} &= \beta_0 \Delta p_{REA,CM,t} + (1-\beta_0)\bar{\pi} && (37)\\ &+\beta_1 \left[p_{REA,Mxn,CON,t-1} - p^*_{REA,Mxn,CON,t-1}\right] && (38)\\ &+\beta_2 \delta_{2009q2} + \epsilon_t\end{aligned}$$

Table 2.8.5: Coefficients and standard errors of the consolidated import deflator

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.22	0.05	0.41	0.05
β_1	0.84	0.17	-0.22	0.11
β_2	-0.22	0.05	-0.03	0.005

$R^2 = 0.65$ (1 step estimation)

Consumption deflator Consumption deflator of the REA is modeled with an ECM equation and estimation in one step from 1996Q4 to 2017Q4. Table 2.8.6 reports the estimated coefficients and Figure 2.8.4 displays a dynamic simulation of the equation.

Target In the long-run, the households' consumption deflator is the average of to the GDP deflator and the consolidated imports, where $\beta_1 = 0.252$ is calibrated using the weight of imports in consumption (see section 2.7 for details). A deterministic trend helps to capture slightly higher consumption price inflation compared to GDP and consolidated imports price inflation within the estimation sample. [28]

$$p^*_{REA,C,t} = \beta_0 + (1 - \beta_1)p_{REA,t} + \beta_1 p_{REA,M,CON,t} + \beta_2 * T_t \tag{39}$$

Short-run equation In the short run, in addition to the error-correction term, the consumption deflator is also determined by GDP deflator and consolidated import deflator and we impose a balanced-growth path condition by anchoring consumption price inflation to the euro area long-run anchor of inflation. Finally, we introduce a short-term non-linear effect of the detrended oil price in euros, to accelerate the transmission of energy price' shocks to consumption price.

$$\begin{aligned}\Delta p_{REA,C,t} \quad = \quad & \beta_0 \left[p^*_{REA,C,t-1} - p_{REA,C,t-1}\right] + \beta_1 \Delta p_{REA,M,CON,t} + \beta_2 \Delta p_{REA,t} \\ & +(1 - \beta_1 - \beta_2)\bar{\pi}_{EA,t} + \beta_3 \Delta \frac{P_{oil,t}}{USD_t \overline{P}_{REA,Y,t}} + \epsilon_t\end{aligned} \tag{40}$$

Table 2.8.6: Coefficients and standard errors of the consumption deflator

	Long-run		Short-run	
	Coef.	s.e.	Coef.	s.e.
β_0	-0.0559	0.0145	0.0583	0.0194
β_1	0.25	-	0.0352	0.0121
β_2	$0.34 \cdot 10^{-3}$	$0.13 \cdot 10^{-3}$	0.7120	0.0793
β_3	-	-	$2.49 \cdot 10^{-4}$	$2.61 \cdot 10^{-5}$

$R^2 = 0.69$ (1 step estimation)

2.9 Financial block

This section deals with the financial block, which in STREAM consists of the short and long government rates.

Short-term interest rate The short rate $i_{EA,t}$ is measured by the 3-month Euribor. In simulation its dynamics are determined by a Taylor rule reacting to euro area inflation and the output

[28] In simulation, in order to preserve the balanced-growth path of the model, this deterministic trend vanishes gradually.

Figure 2.8.4: REA consumption deflator

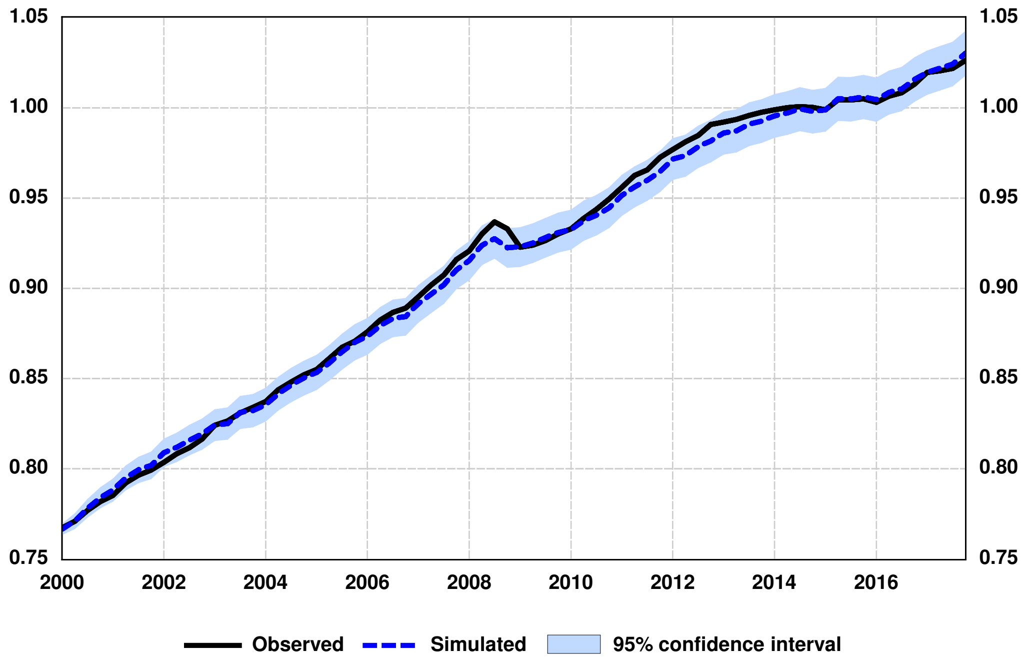


Table 2.9.1: Variables used in section 2.9

Notation	Description
$i_{EA,t}$	3-month Euribor
$i_{REA,10,t}$	10-year sovereign bond rate for REA
$s_{REA,10,t}$	Term spread of the 10-year REA government bond

gap, given by (41), the E-SAT core equation for the short rate. While in principle the euro area inflation and output gap are determined in STREAM by aggregating France and the rest of the euro area as a output-weighted average, in practice when STREAM is operated on its own these quantities only account for REA dynamics, as French inflation and output gap are exogenous.[29]

$$(1 - \lambda_i L) \left(i_{EA,t} - \bar{\imath}_{EA,t}\right) = (1 - \lambda_i) \left(\alpha_i \left(\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}\right) + \beta_i \hat{y}_{EA,t-1}\right) + \varepsilon_t \tag{41}$$

Variation in the short rate is the primary driver of financial variables' dynamics in STREAM. Even though the short rate itself does not appear in any of the main behavioral equations, it determines the dynamics of the REA 10-year rate, which in turn either affects the real sector directly or is a key determinant of other interest rates (e.g. the user cost of capital for firms). Furthermore, as the short rate is a core component of the E-SAT expectations model, it has an

[29]See section 3 for details on the construction of a two-country model where the euro area output gap and inflation are completely endogenous.

effect on agents' behavior via all the expectation terms in the backward-looking setup.

As in the E-SAT model, the long-run anchor of the short-term interest rate $\bar{\imath}_{EA,t}$, measured by the 5-year ahead forward rate of the 3-month Euribor rate, is simply modeled as an AR(1) process. As this process is exactly the same as in FR-BDF, we refer the interested reader to (Lemoine et al., 2019) for details.

Long-term government interest rate The REA long rate is approximately measured as the output-weighted average return on 10-year government bonds issued by Germany, Italy, Spain and the Netherlands. It plays an important role in STREAM as the foundation for rates paid by the private sector, particularly the user cost of capital $r_{K,REA,t}$. The household consumption decision is also affected by this rate.

Its dynamics are determined by the term structure equation (42) which relates the 10-year rate to an expectation component $\mathrm{PV}\left(i_{EA}\right)_{t|t-1}$ and the term spread $s_{REA,10,t}$, which we assume to follow a simple $AR(1)$, as per (43). The estimation results are presented in Table 2.9.2. It shows in particular that the steady state of the term premium is estimated equal to 0.7% in annualized percentage points $(400 * 1.77 \cdot 10^{-3})$. $\mathrm{PV}\left(i_{EA}\right)_{t|t-1}$ is determined using E-SAT; the relevant policy function is described in Table 2.9.3.[30] Figure 2.9.1 plots the elements of equation (42). The downward trend of the long-term interest rate seen in the plot is captured by the expectation component, which is tightly driven by market-based long-run expectations $\bar{\imath}_{EA,t}$.

$$i_{REA,10,t} = \mathrm{PV}\left(i_{EA}\right)_{t|t} + s_{REA,10,t} \tag{42}$$

$$s_{REA,10,t} = \left(1-\rho_{10}\right)\bar{s}_{REA,10} + \rho_{10} s_{REA,10,t-1} \tag{43}$$

The term structure equation (42) has its theoretical foundations in an approximation where the bond is modeled as having an infinite maturity with coupon payments that decay at a geometric rate. The calibration of the decay is chosen so that the distance between our approximated bond and the 10 year bond is minimized. The implied theoretical equation for $i_{REA,10,t}$, the yield at maturity of a hypothetical long term bond, is then

$$i_{REA,10,t} = \left(1-\kappa_{10}\right)\sum_{s=0}^{\infty}\kappa_{10}^{s}\left(i_{EA,t+s}\right) + s_{REA,10,t} \tag{44}$$

where κ_{10} is the decay – linked to the duration of the bond – and $s_{REA,10,t}$ is the theoretical term spread.

2.10 Accounting framework and public finances

Compared to choices made for the accounting framework of FR-BDF, we do for the one of STREAM the following simplifications:

[30] Note that no additional auxiliary equation is needed as $i_{EA,t}$ is included in the E-SAT core.

Figure 2.9.1: REA long-term government interest rate and its components, annualized percent

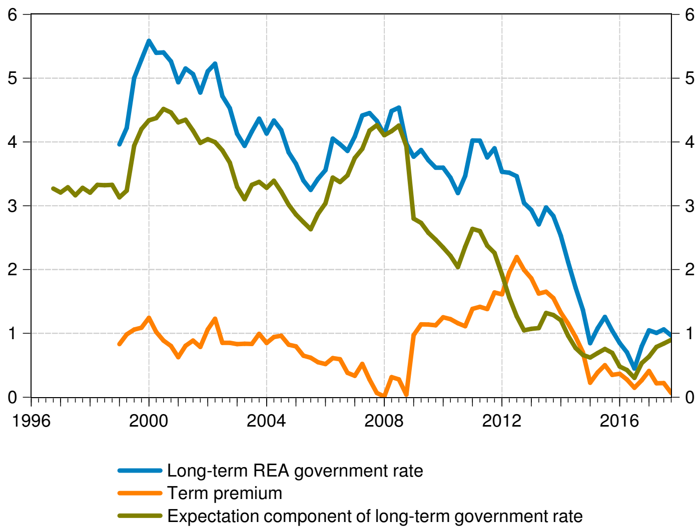


Table 2.9.2: Coefficients and standard errors, term structure equation

Coef.	Estimate	s.e.
ρ_{10}	0.94	0.05
$\bar{s}_{REA,10}$	$1.77{\cdot}10^{-3}$	$9.75{\cdot}10^{-4}$

- we group business investment of corporated and unincorporated firms, household investment, government investment and inventory changes within a variable named here total investment;
- we also group consumption of government and of non profit institutions serving households (NPISH) within a variable named here public consumption;
- we do not model accounts of agents, except a simplified expenditure-receipt account of the government (see below);
- we model total economy variables, instead of distinguishing market from non-market branches;
- we model accounting identities for sums of volumes of variables with constant-price aggregations augmented with residuals, instead of chained-price aggregations.

The government block is here a simplified expenditure-receipt account and we adopt the following common principles on receipt and spending sides:

- On the receipt side, we distinguish direct income taxes paid by households from other receipts, each receipt being determined by an exogenous effective tax rate and on an endogenous tax basis (respectively GDP and disposable income of households);

Table 2.9.3: Coefficients of the policy function for the expectation of the 3-month bond rate

VAR model	Coefficient
Constant	0.003
$\hat{y}_t$	0
$i_{EA,t}$	0.16
$\bar{i}_{EA}$	0.51
$\pi_t - \bar{\pi}$	0
$\hat{y}_{EA,t}$	0.026
$\pi_{EA,t} - \bar{\pi}_{EA,t}$	0.041

- On the spending side, we distinguish public consumption, social transfers and other spending, the ratio of their volume relative to long run output being assumed to be exogenous.

One exception to these principles concerns social transfers $T_{G,t}$. As is the case for other variables on the spending side, their volume is related to long run output through a ratio, $\tau_{TG,t}$, but this ratio is endogenized with equation (45) to ensure the convergence of the government's balance ratio toward its long run target:

$$\begin{aligned} \tau_{REA,TG,t} &= (1-\rho_{stab,1})\,\tau_{REA,TG,t-1} + \rho_{stab,1}\tau^*_{REA,TG} \\ &\quad -\rho_{stab,2}\left(\frac{-B_{REA,G,t}}{Y_{REA,t}P_{REA,Y,t}} - \frac{\exp(g+\bar{\pi})-1}{\exp(g+\bar{\pi})}\frac{-W_{REA,G}}{Y_{REA}}\right) \end{aligned} \tag{45}$$

where $B_{REA,G,t}$ is the net financing capacity of government, $\frac{W_{REA,G}}{Y_{REA}}$ is the long-run target of the ratio of net financial assets of government over nominal GDP and $\tau^*_{REA,TG}$ is the long-run target of the ratio of social transfers over nominal GDP. We calibrate the net asset target $\frac{W_{REA,G}}{Y_{REA}}$ at the same value as for France, i.e. at 40% in percent of annual GDP. We calibrate the transfer ratio $\tau^*_{REA,TG}$ at 3.9%, a level ensuring the budget constraint of the government in the long run for a government deficit consistent with this net asset target.[31] We calibrate both stabilization parameters $\rho_{stab,1}$ and $\rho_{stab,2}$ at 0.1.

3 Combining FR-BDF and STREAM to get EA-BDF

One possible use of the STREAM model is to combine it with the FR-BDF model into a two-country model, which we call the Euro Area model of Banque De France (EA-BDF). We model with FR-BDF and STREAM the two endogenous economies of this model, France and the rest of the euro area, while the world outside the euro area is assumed exogenous. This model can then be used to study for example economic questions that require fully endogenous euro area monetary policy – in the sense that the policy simultaneously accounts for economic developments both in

[31] This value is computed through a numerical resolution of the balanced growth path.

France and elsewhere in the euro area – or questions that concern interdependence between the two. This section describes this procedure of combining these models, from the perspectives of economic theory and technical implementation, which are to some extent interconnected.

In a nutshell, this new model is constructed simply by taking the equations describing the two models and combining them as appropriate, see Figure 3.0.1. First, for the vast majority of equations, belonging to the core of both models and not to their foreign blocks, no changes or adaptations are necessary, e.g. French business investment and REA household consumption are determined using exactly the same equations as in the separate models. Second, we need to add some extra equations for connecting trade variables of both models through intra components of world demand and competitor prices. Third, some equations are such that they appear in both FR-BDF and STREAM – this is the so called foreign block of the two models. On this side, we design a common financial block, by taking care of relating monetary policy to aggregated developments of the euro area and to model the euro area real effective exchange rate in relation with its equilibrium level.

3.1 Trade linkages

On top of including core equations of FR-BDF and STREAM, we include in EA-BDF extra equations for endogenizing intra components of world demand and of price of competitors of both endogenous blocks of the model. As shown below, these equations are simple bridge equations, which relate these variables to the import volume and the export price of its euro area trade partner. For the REA, the intra component of world demand is related to the import volume of France and their intra prices of competitors of both related to the export price of France (on export and import sides). For France, we specify such equations in a symmetric way. These bridge equations include exogenous rates ($\tau_{REA,WD,IN,t}$, $\tau_{REA,CX,IN,t}$, $\tau_{REA,CM,IN,t}$, $\tau_{WD,IN,t}$, $\tau_{CX,IN,t}$ and $\tau_{CM,IN,t}$) which we keep constant in simulations.

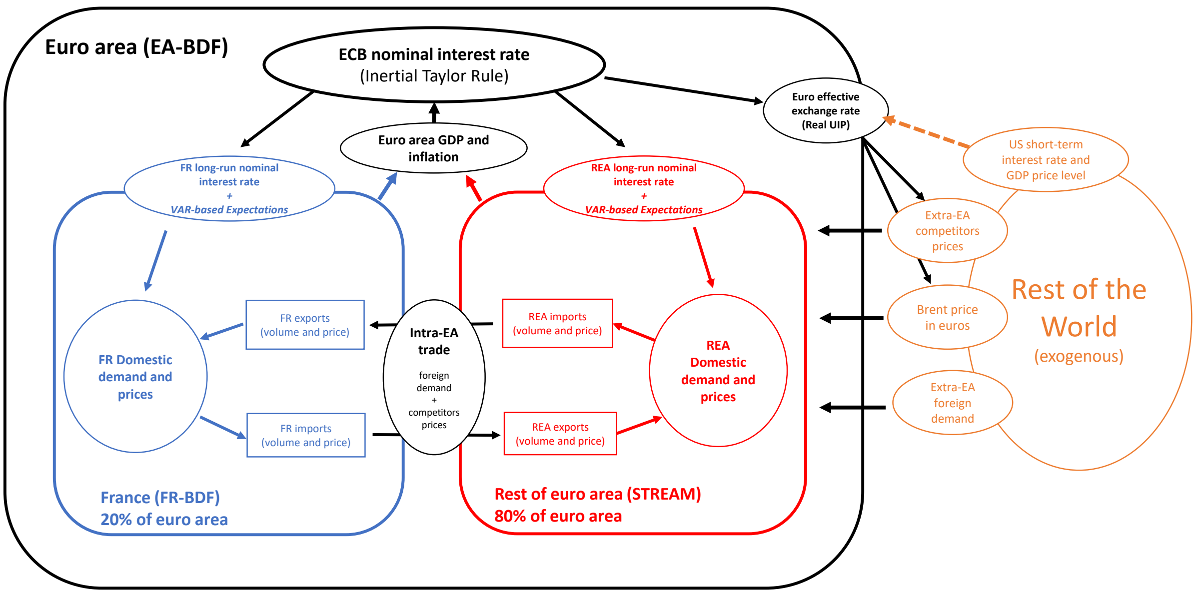
$$\begin{aligned}
WD_{REA,IN,t} &= \tau_{REA,WD,IN,t} M_t && (46)\\
P_{REA,CX,IN,t} &= \tau_{REA,CX,IN,t} P_{X,t} && (47)\\
P_{REA,CM,IN,t} &= \tau_{REA,CM,IN,t} P_{X,t} && (48)\\
WD_{IN,t} &= \tau_{WD,IN,t} M_{REA,t} && (49)\\
P_{CX,IN,t} &= \tau_{CX,IN,t} P_{REA,X,t} && (50)\\
P_{CM,IN,t} &= \tau_{CM,IN,t} P_{REA,X,t} && (51)
\end{aligned}$$

3.2 Euro area variables and monetary policy

In FR-BDF and STREAM, monetary policy is determined by a standard Taylor rule of the form:

$$(1-\lambda_i L)\left(i_{EA,t}-\bar{\imath}_{EA,t}\right) = (1-\lambda_i)\left(\alpha_i\left(\pi_{EA,t-1}-\bar{\pi}_{EA,t-1}\right)+\beta_i \hat{y}_{EA,t-1}\right)+\varepsilon_t \qquad (52)$$

Figure 3.0.1: Comprehensive scheme of EA-BDF



where $\pi_{EA,t-1}$ and $\hat{y}_{EA,t-1}$ are euro area inflation and output gap, respectively. Though the form of the monetary policy rule is the same in EA-BDF, the computation of these quantities has to be adapted to the new model. While they are partly or fully exogenous in the two smaller models, in EA-BDF we instead compute them as weighted averages:

$$\hat{y}_{EA,t} = \omega \hat{y}_t + (1 - \omega)\hat{y}_{REA,t} \tag{53}$$

and

$$\pi_{EA,t} = \omega \pi_t + (1 - \omega)\pi_{REA,t} \tag{54}$$

where ω is the share of French output to total euro area output, estimated at 21%, so that in EA-BDF the short rate is set taking into account the economic developments of the whole euro area.

3.3 Equation of the euro real effective exchange rate

We obtain the EA-BDF equation of the euro real effective exchange rate (REER) through several steps. First, ξ_t being the log of the euro nominal effective exchange rate (NEER), expressed in direct quotes, we model the log of the euro REER $q_t = \xi_t + p_{EA,t} - p_{F,t}$ with an uncovered interest rate parity (UIP) condition with respect to the foreign block (rest of the world).[32]

$$q_{EA,t} \quad = \quad E_t\left(q_{EA,t+1}\right) + \left(i_t - E_t(\pi_{EA,t+1})\right) - \left(i_{F,t} - E_t(\pi_{F,t+1})\right)$$

Second, we solve this UIP condition with forward iterations, in order to relate the euro REER with the present value of real interest rate differentials. In such a solved form, the euro REER should also depend on its long-run value $\bar{q}_{EA}$.

$$q_{EA,t} \quad = \quad \bar{q}_{EA} + \left(PV_{nd}(i)_{t|t} - PV_{nd}(\pi_{EA})_{t+1|t}\right) - \left(PV_{nd}(i_F)_{t|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\right)$$

where i_t, $i_{F,t}$, $\pi_{EA,t}$ and $\pi_{F,t}$ are the euro area and foreign price level, interest rate and inflation, respectively, and $PV_{nd}(x)_{t|t-1}$ is the non-discounted present value for some variables x among these four variables.[33] Notably $\bar{q}$ would not be a free parameter, but would be the unique steady state of the euro REER implicitly pinned down by the whole model. We provide in the next subsection some intuition about the existence of such a unique steady state within the stylized small open economy framework of Galí & Monacelli (2005).

Third, given the large size of the EA-BDF model, an analytical expression of this euro REER steady state as a function of parameters of the model is not available. Hence, as a workaround, we use in EA-BDF a modified condition (55), where, compared to the former equation, we have replaced the steady-state REER $\bar{q}_{EA}$ by a time-varying equilibrium REER $\bar{q}_{EA,t}$. We model this equilibrium REER $\bar{q}_{EA,t}$ with a learning rule (56), which updates its value each period by 5 percent of the gap between the ex post REER and the prior estimate of the equilibrium REER. This learning rule ensures that the equilibrium REER converges in the long run toward to steady-state REER implicitly pinned down by the model.

$$q_{EA,t} \quad = \quad \bar{q}_{EA,t} + \left(PV_{nd}(i)_{t|t} - PV_{nd}(\pi_{EA})_{t+1|t}\right) - \left(PV_{nd}(i_F)_{t|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\right) \quad (55)$$

$$\bar{q}_{EA,t} \quad = \quad \bar{q}_{EA,t-1} + (1-\gamma)\left(q_{EA,t-1} - \bar{q}_{EA,t-1}\right) \quad (56)$$

where γ is calibrated at 0.95.

3.4 Long-run determination of the real exchange rate in a stylized model

In order to understand why the home economy's real exchange rate is uniquely pinned down in the perfect foresight steady state, given that the euro area of our model is a small open economy (it does not influence the rest of the world), we can make a parallel with the stylized framework of Galí & Monacelli (2005), where the home country and foreign countries would correspond to the euro area and the rest of the world. For simplifying purposes, they assume symmetry among all countries (other than the home country), and then show how the real exchange rate and output in the home

[32] With respect to data, we proxy here all foreign variables with United States time series.

[33] See Lemoine et al. (2019) for details on the computation of these present value variables.

economy are determined. Without loss of generality, they assume a unit value for productivity in all foreign countries, and a productivity level A in the home economy. They show that, in the symmetric case (when $A = 1$), the real exchange rate of the home economy must necessarily be equal to unity in the steady state, whereas output in the home economy coincides with that in the rest of the world.

For obtaining this result, first, they show with the clearing condition combined with the international risk sharing condition that the demand of home output Y, on top of being driven by foreign output Y^*, also depends in a positive way of the real exchange rate S, i.e. that a depreciation of the real exchange rate boosts the demand for home output:

$$Y = v(S)Y^*,$$

where $v(S) > 0, v^{'}(S) > 0$ and $v(1) = 1$.

Second, they show with the labor supply condition combined with the international risk sharing condition that the supply of home output, on top of being also dependent on foreign output, depends in a negative way on the real exchange rate:

$$Y = \left(\frac{1 - 1/\epsilon}{(1-\tau)\,(Y^*)^{\sigma}\,S} \right)^{1/\phi},$$

where $\epsilon > 1$ denotes the elasticity of substitution between varieties produced within any given country, ϕ denotes the inverse of the Frisch elasticity of the labor supply, σ denotes the risk aversion of households and τ denotes a constant employment subsidy that neutralizes the distortion associated with firms' market power.

Finally, these demand and supply conditions pin down a unique steady state $S = 1$ for the real exchange rate, as well as the steady state of home output, equal to the one of foreign output.

4 Impulse responses of EA-BDF

In this section, we show impulse response functions (IRFs) of EA-BDF, both for the rest of euro area and France. We focus on three shocks that summarize the main properties of our two-country model: a short-term interest rate shock, a foreign demand shock and a cost-push shock. In appendix, we also provide IRFs for two additional shocks: a term premium shock to long-term rate and a risk premium shock to nominal exchange rates.

We run an unconditional simulation from 2018Q1 to 2250Q1 which is our baseline, then we re-run an alternative simulation from 2150Q1 to 2250Q1 with the shock hitting the economy in 2150Q1. Exogenous variables are extrapolated with the three key growth rates that prevail along the balanced growth path: $\Delta\bar{y}$ for real variables homogenous to output, $\Delta\bar{e}$ for real variables homogenous to labor productivity and $\bar{\pi}$ for nominal variables homogenous to a price. Trends are modeled with backward-smoothing equations; see section 4.9 in Lemoine et al. (2019) for details. IRFs are then calculated as percentage or absolute deviations, depending on the type of variable, between the alternative and baseline scenarios.

Finally, we perform our simulations under both VAR-based (VBE) and model-consistent expectations (MCE). In VAR-based simulations, agents' expectations are backward-looking and based on E-SAT; in model-consistent simulations, agents are forward-looking and form their expectations based on the model forecasts under perfect foresight.

4.1 Short-term interest rate shock

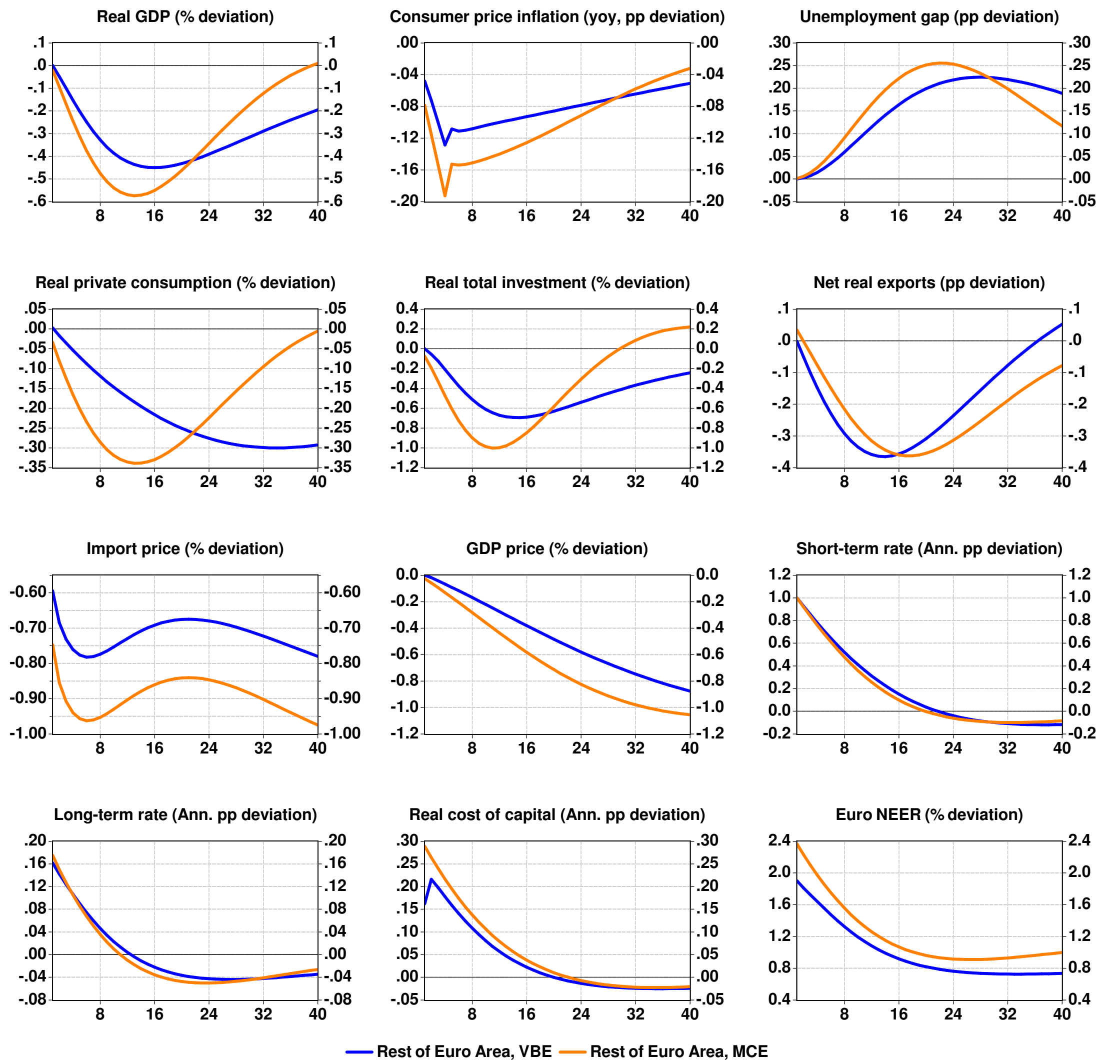
We simulate a one-period +100bp shock to the annualized short-term interest rate which is endogenously passed on to the long-term rate through the term structure equation and to the nominal effective exchange rate through the UIP. The shock is transmitted through the inertial Taylor rule with persistence $\lambda_i = 0.92$. Figures 4.1.1 and 4.1.2 respectively show the impulse responses functions of REA and FR.

For both the rest of euro are and France in Figures 4.1.1 and 4.1.2, whatever the type of expectations, the monetary policy shocks transmits to the economy through three channels: (i) nominal exchange rates, (ii) long-term interest rates and cost of capital and (iii) expected permanent income as well as all other expected variables. First, net real exports decrease after the nominal exchange rate appreciation. Second, on the real total investment side, raising real cost of capital depresses investment demand. Third, on the private consumption side, households expect a decrease of permanent income and decrease consumption. In the medium run, downward adjustment in prices and deflators improves external price-competitiveness. Also, long-term interest rates decrease because of monetary policy accommodation, due to negative output gap and lower inflation. These forces push back demand components toward their baseline level.

Both types of expectations matter in different ways for financial and non-financial variables. Regarding expectations of financial variables, MCE amplifies responses compared to VBE, notably for the nominal effective exchange rate. In contrast, endogenous responses of short-term and long-term interest rates are quite closer to each other in VBE and MCE cases, the persistence of the shock being mainly driven by the inertia of the Taylor rule used both in EA-BDF and in E-SAT. As expectations regarding the short-term interest rate are similar in both cases, differences regarding the response of the real exchange rate are largely due to differences in inflation expectations. As inflation is more persistent in EA-BDF than in E-SAT, the fall of expected inflation is larger under MCE as well as the increase of expected real interest rates and this generates a stronger appreciation of the real effective exchange rate in the short run. Given the stickiness of prices, we also get a stronger appreciation of the nominal effective exchange rate in the short run. The larger responses of both internal and external demand components of GDP in MCE compared to VBE are due to the larger nominal exchange rate appreciation. The stronger response of the exchange rate, in turn, will explain the larger drop in net real exports and then other demand components through second-round effects.

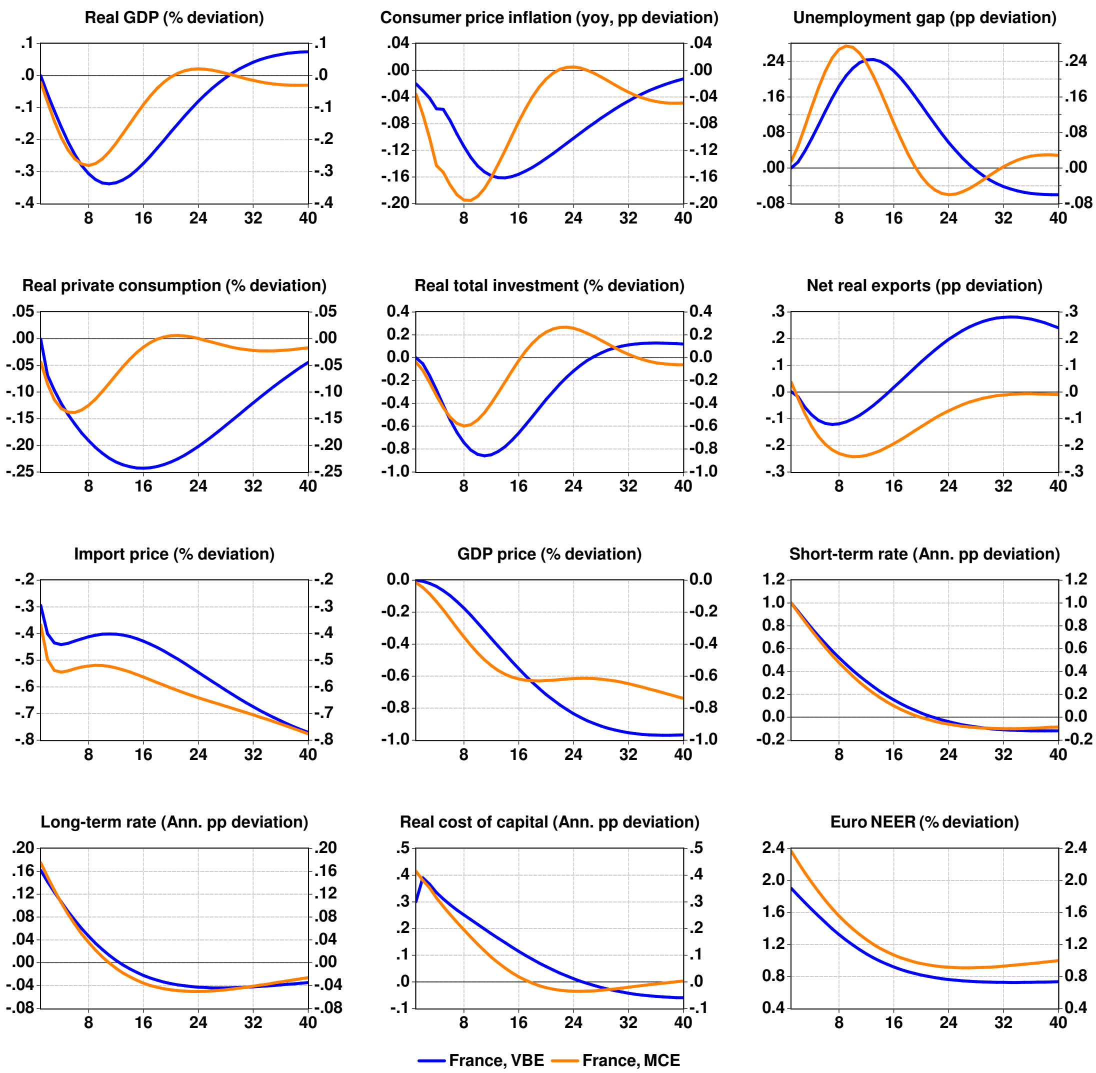
Regarding expectations of non-financial variables, MCE fastens and dampens the transmission compared to VBE. With respect to the speediness of the transmission, under MCE, the shock im-

Figure 4.1.1: Short-term interest rate shock, IRFs for the REA



mediately affects all components of demand and price, because forward-looking agents immediately adjust their behavior (see for example the response on impact under MCE of consumption, investment and the consumption price). In contrast, the shock is delayed under VBE and the shock gradually transmits through domestic demand component. With respect to the strength of the response, two differences stand out, which both dampen responses under MCE compared to VBE. First, the peak response of the real cost of capital is smaller under MCE than under VBE, because of the higher persistence of inflation in EA-BDF than in E-SAT already discussed above. A smaller increase of the real cost of capital implies a smaller fall of investment and, hence, dampens the fall of GDP. Second, in the French case, the fall of consumption is weaker under MCE than under VBE, because of a similar feature of the response of permanent income caused by a smaller persistence of income dynamics in EA-BDF than in E-SAT.

Figure 4.1.2: Short-term interest rate shock, IRFs for France

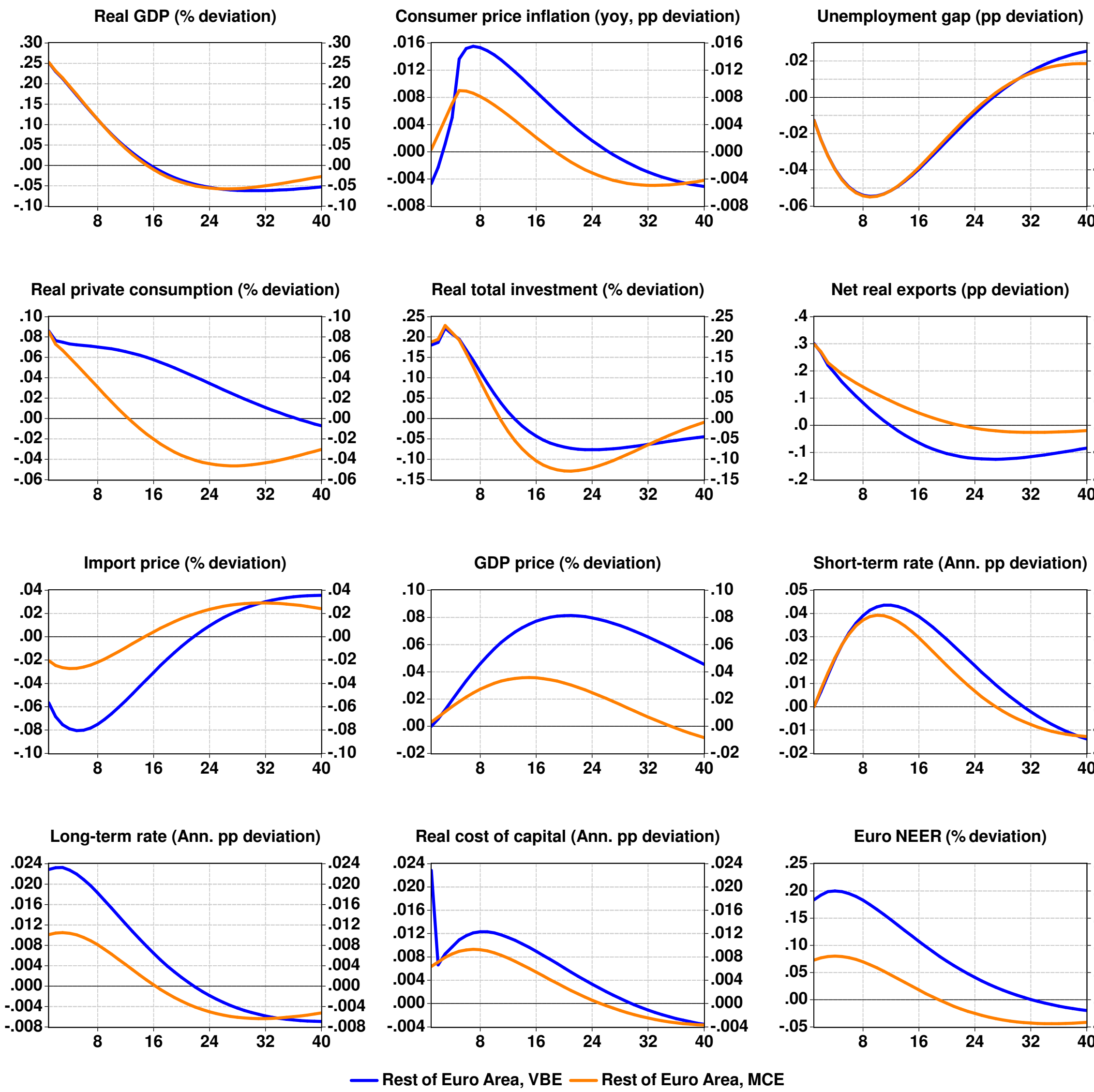


4.2 Foreign demand shock

The foreign demand shock is a *symmetric* +1% shock to the volume of extra-EA foreign demand addressed to the rest of euro area and France exporters, with an *ad hoc* persistence $\rho = 0.9$. Figures 4.2.1 and 4.2.2 show the impulse responses functions respectively for REA and France.

First, the shock has broadly similar effects in France and REA. On the real side, the short-run increase in external demand stimulates real private consumption and total investment through the effect on GDP and households' permanent income. On the nominal side, the lower unemployment and higher output gap boost domestic inflation rates in both regions, to which EA monetary policy reacts by raising the short-term interest rate. After two years, the economy starts to stabilize due to the fading of the extra-EA foreign demand shock and also to price-competitiveness losses from

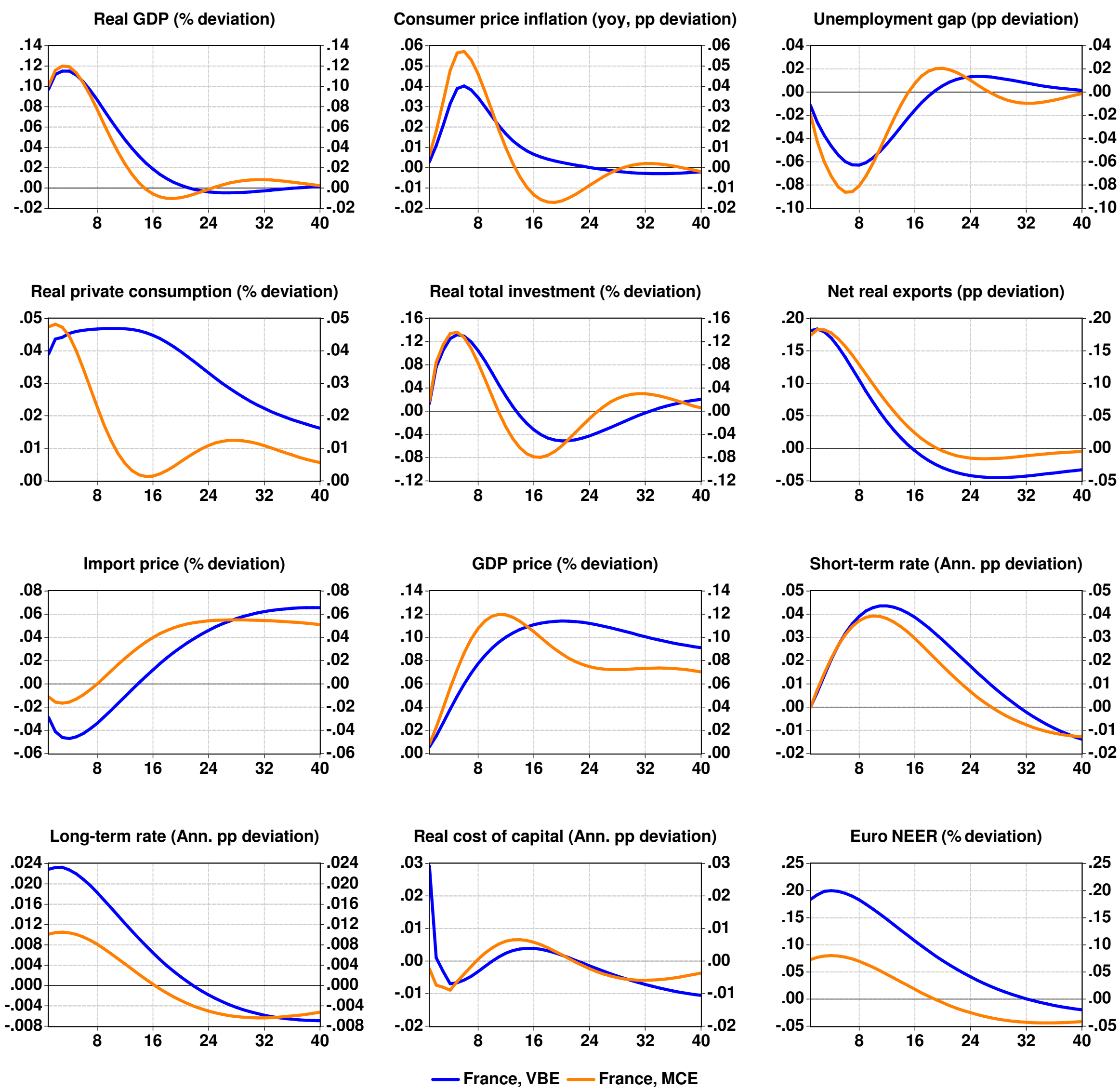
Figure 4.2.1: Foreign demand shock, IRFs for the REA



higher domestic inflation and the appreciation of the nominal effective exchange rate. A notable difference between REA and France is related to the response of the real user cost of capital, which is increasing in the REA while rather decreasing in France. As this difference is a general property of our EA model in response to aggregate demand shocks, we will discuss it in section 5.1 when commenting the impulse response to a symmetric government consumption shock.

Second, differences between VBE and MCE cases are relatively limited in terms of real GDP, which is not a surprise as both FR-BDF and STREAM trade blocks are not modeled using PAC equations or depend on expectations. Differences concentrate on private consumption, the GDP price and financial variables. First, households' consumption faster decreases in the MCE case than

Figure 4.2.2: Foreign demand shock, IRFs for France

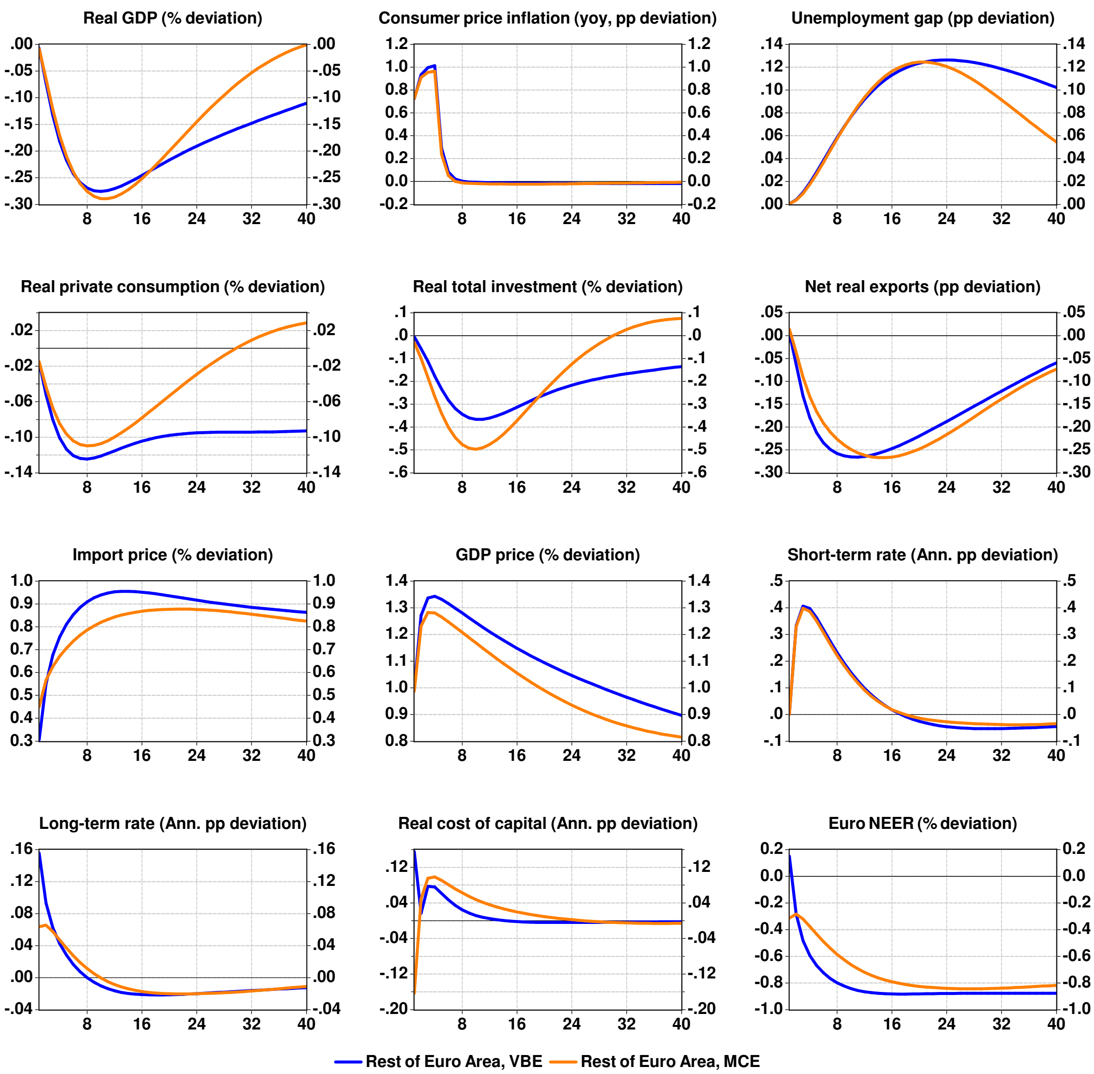


in the VBE case, as forward-looking agents expects the income response to the shock to be less persistent. Second, on the nominal side, we observe lower effects (even negative at some point) on inflation in the medium run in the MCE case. Third, as a result, monetary policy is less tightened: the short-run interest rate is lower and both the nominal exchange rate appreciation and the increase in long-term interest rate are smaller in the short run. While the smaller response of prices, combined with a lower euro appreciation, brings a smaller improvement of terms of trade and hence likely dampens the response of private consumption, the weaker appreciation of the euro increases the response of net real exports.

4.3 Cost-push shock

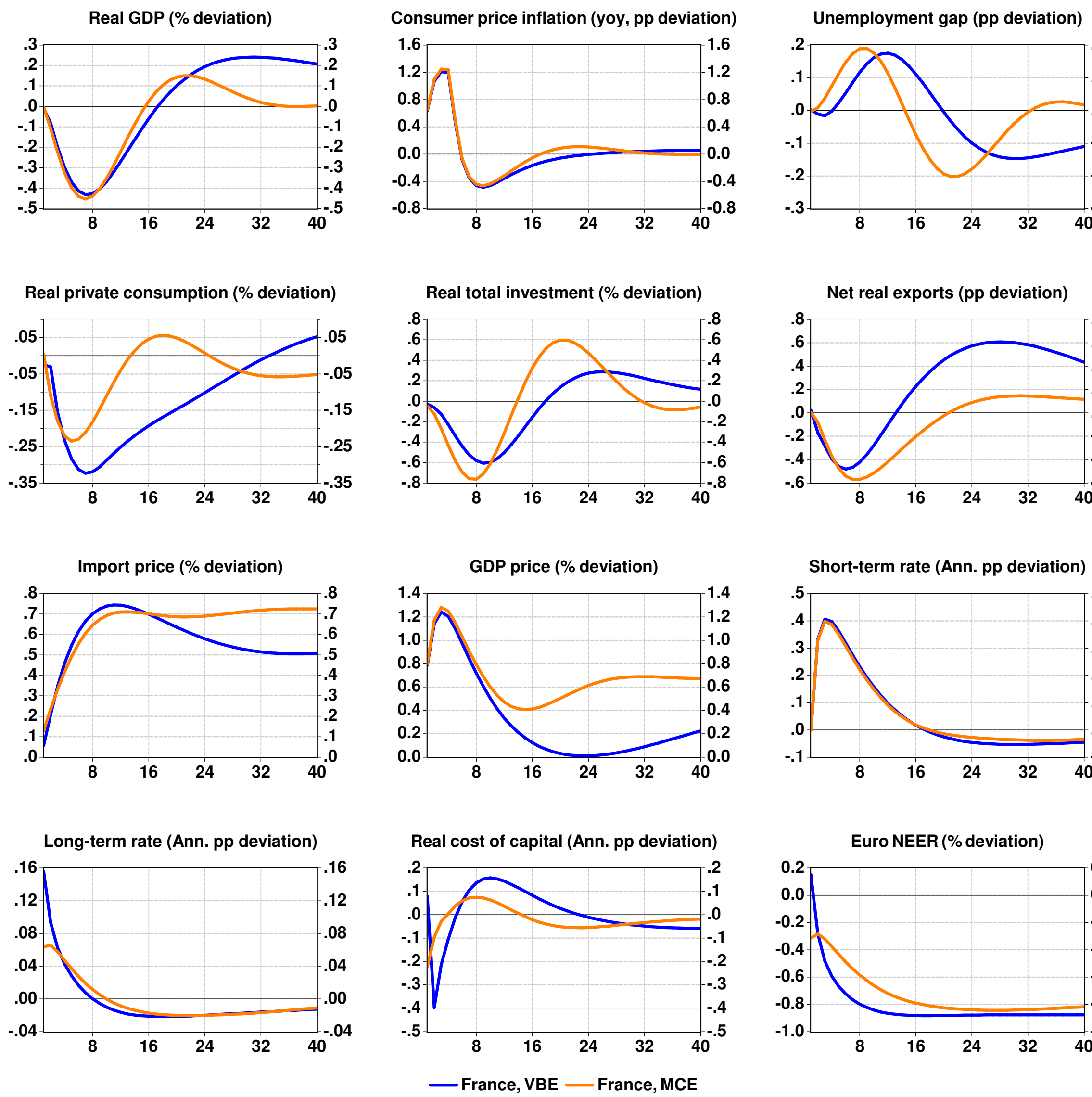
The cost-push shock is defined as a *symmetric* $+1\%$ shock to the annual inflation rate in both France and the rest of euro area. For France in FR-BDF, the shock initially transmits through the factor price frontier equation for the market branches value-added deflator; for the rest of euro area in STREAM, the shock passes through the New-Keynesian Phillips curve. Figures 4.3.1 and 4.3.2 show the impulse responses respectively for REA and France.

Figure 4.3.1: Cost-push shock, IRFs for the REA



In the short run, the shock endogenously passes to the rest of the model through exports' and consumption price deflators. First, because of the increase of the export deflator, the real effective exchange rate appreciation deteriorates external competitiveness and weights on real exports and external demand. Second, higher consumer price inflation decreases the purchasing power of households and real consumption. As a second-round effect, lower aggregate demand depresses real investment. In reaction to higher inflation, monetary policy increases the short-term nominal interest rate, which results in a higher long-term interest rates, although less than expected inflation implying lower real user cost of capital. The latter dampens the negative response of real total investment in the short run. In the medium run, inflation falls below baseline and domestic and export prices gradually decrease, restoring external price-competitiveness in both regions, although with quantitative differences.

Figure 4.3.2: Cost-push shock, IRFs for France



While responses of French and REA block are broadly, qualitatively similar in the short run, the French economy displays a faster but also more cyclical convergence than the REA economy, both in VBE and MCE cases. First, we observe that real effects of the cost-push shock are larger for the French economy for both domestic and external demand components. Second, comparing impulse response of domestic prices between France and REA, we observe that French domestic prices tend to adjust faster after the shock. In addition to larger effects in the short-run, the steeper Phillips curve in France (see section 2.4) compared to the rest of euro area further explains why prices adjust faster in the French economy and helps restoring external price competitiveness. Trade spillovers from REA to France are quantitatively important, especially regarding the response of exports and imports prices and net exports. First, the increase in REA exports' price attenuates the degradation of terms of trade for France and dampens the response of French imports. Second, the euro nominal effective exchange rate depreciates and further dampens the loss of price-competitiveness and the drop in real exports.

Finally, differences related to expectations are limited. On impact, we observe a transitory appreciation in the VBE case, due to expectations of future interest rate hikes from monetary policy, while the nominal exchange rate immediately depreciates in the MCE case, due to expectations of a higher future price-level. Regarding demand components, only real consumption and, to a lesser extent, real investment display a dampened response to the shock when agents are forward-looking compared to the VBE case.

5 Government spending multipliers in a monetary union

In this section we study the effects of the government spending shock in a monetary union with the EA-BDF model. First, in the current context of a widespread government spending stimulus in the whole EA, we ask what is the size of the government spending multiplier in France depending on the symmetry/asymmetry of the shock at the EA level. In the symmetric case, expansionary effect in France are potentially reinforced by trade spillovers, but they might be counteracted by the monetary policy response. Our second question concerns the size of the government spending multiplier depending on the monetary policy rule. We consider three alternative rules: inflation targeting, price-level targeting and average-inflation targeting. And finally, both questions are regarded through the lens of a particular expectation formation: backward-looking expectation vs model-consistent one. All simulations are run around a baseline at a date far in the future, in order to be at the balanced-growth path.

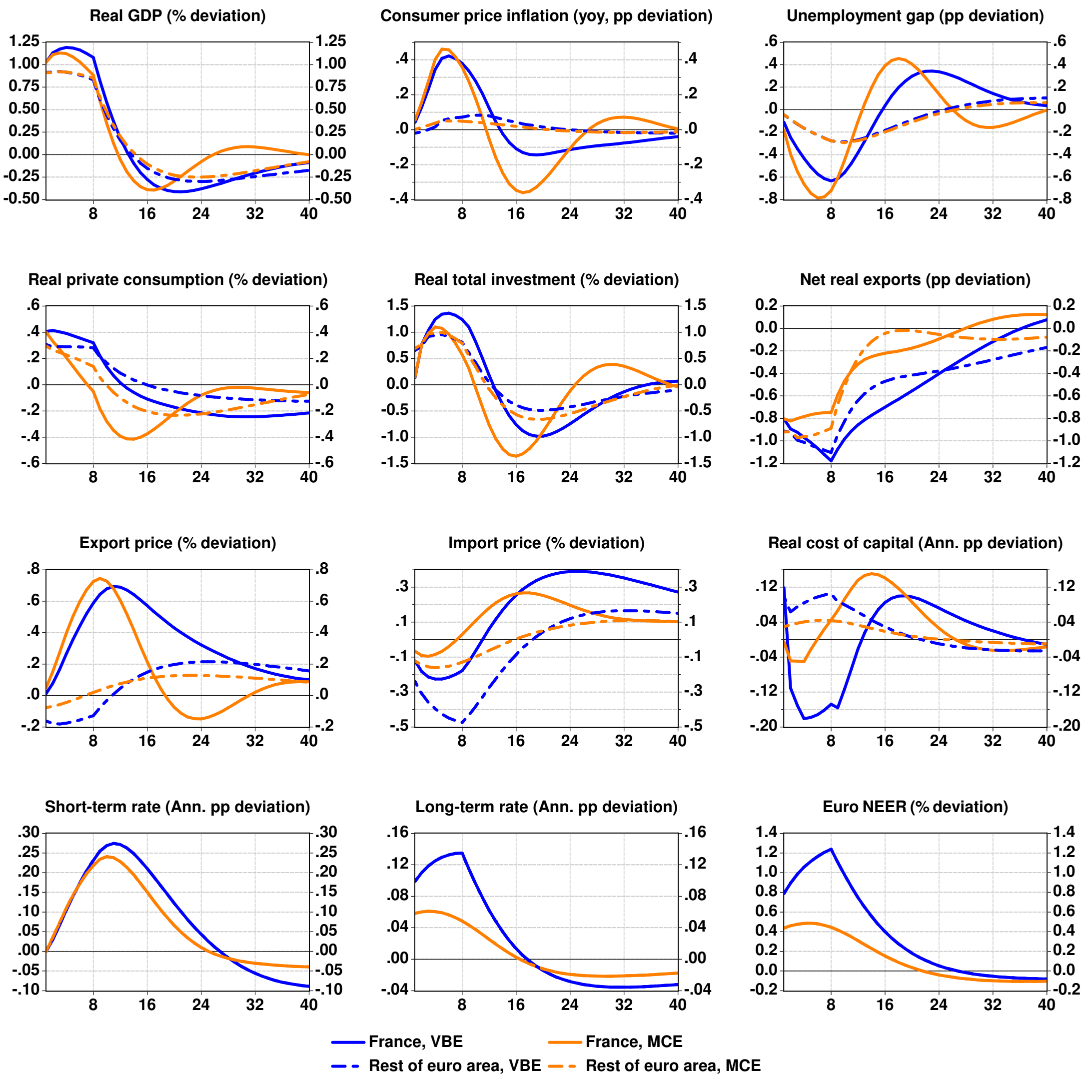
5.1 Symmetric shock

Using the EA-BDF model, we simulate a +1 pp of long-run GDP *symmetric* shock to real government consumption in the rest of euro area and in France, kept constant for 2 years and with calibrated persistence $\lambda_g = 0.8$ afterward. In each region, government stabilizes public deficit-to-GDP and debt-to-GDP ratio through an inertial fiscal rule.[34] These fiscal rules are non-aggressive

[34]See section 2.10 and section 4.8.5 in Lemoine et al. (2019) for details about the fiscal rules of STREAM and FR-BDF.

as they stabilize public debt in a very gradual manner by targeting the debt-stabilizing total deficit. As a result, the shock is largely debt-financed in the short run. Figures 5.1.1 shows VBE and MCE impulse responses for REA (dashed lines) and France (solid lines).

Figure 5.1.1: Government consumption shock, IRFs for the REA and France



The propagation of the shock is relatively similar in the REA and France for both types of expectations. In response to higher inflation and positive output gap in the short run, monetary policy increases the short-term interest rate through the inertial Taylor rule. Long-term and nominal exchange rates increase on impact due to expectations of future higher nominal interest rate. In the short-run, the shock to government real consumption directly increases aggregate demand and GDP, which then stimulates private consumption and investment. The positive effect on domestic demand is partially counteracted by the decrease of net real exports: higher domestic demand boosts imports while the nominal exchange rate appreciation reduces external price competitiveness and real exports. In the medium run, the shock on government real consumption gradually fades out. Lower aggregate demand and higher real interest rates reduce domestic demand components. On the one hand, fiscal consolidation gradually reduces household permanent income and private consumption decreases and, on the other hand, higher real interest rates combined with lower output also depress private investment and, to a limited extent, household consumption. In response to lower inflation and negative output gap, monetary policy endogenously decreases the nominal interest rate through the Taylor rule. Finally, the nominal exchange rate depreciates, gradually restoring external price-competitiveness and net real exports increases.

On impact, the multiplier is only slightly smaller in the REA compared to France. While in France on average during the first two years the multiplier is around 1.1 in VBE case and 1 in MCE case, for the REA the multiplier is 0.9 for both types of expectations. To understand the difference in multipliers across different regions (France vs REA) we shall analyze the response of the real cost of capital after the government consumption shock: in the REA we observe an increase in the real cost of capital while in France particularly in the VBE case there is a substantial fall for 2.5 years starting from the second quarter after the shock. Given that the Taylor principle holds, the response of the real user cost of capital for the REA is quite intuitive: the increase in aggregate demand boosts both GDP and inflation, monetary policy raises its policy rate to stabilize the economy, which raises the long-term interest rate and the cost of capital. In the case of France, however, the increase of the long-term rate is lower than inflation expectations at the beginning (except for the first quarter). First, and most importantly, the smaller response of interest rate to French inflation developments is related to the small share of France within the euro area (21%). Second, the response of inflation is larger in France than in REA the due to a steeper Phillips Curve slope in France (see section 2.4 for more details). As a result, the government consumption shock has an additional positive effect on investment through inflation expectations and real cost of capital in France, but a negative one in the REA.

Having compared the multipliers across regions, we now turn to the comparison across different types of expectations. Although real GDP response is relatively close in VBE and MCE, we observe differences for sub-components of aggregate demand and prices. First, consumption falls back faster below its baseline level, due to expectations of future fiscal consolidation in the MCE case. This can be interpreted as households being more Ricardian in MCE compared to VBE, essentially because they are forward looking and fully expect future fiscal consolidation and its effect on permanent income. In a similar manner, real investment decreases more in the MCE case due to higher real interest rate; although the long-term nominal interest rate increases less in the short run, expected inflation is also lower in that case. Second, on the external demand side, net real exports restore faster in the MCE case. While real imports behave relatively similarly in the short run (due to similar response of domestic demand), real exports decrease less under MCE, because of the smaller nominal exchange rate appreciation. Third, even if real GDP and unemployment behave closely, especially in the REA, the positive response of inflation is more short-lived in the MCE case.

There are two types of spillovers from REA on the French economy. On the one hand, we have positive spillovers from trade due to higher foreign demand from the REA, which dampens the negative impact of exchange rate on real exports. And on the other hand, there are negative spillovers from higher nominal interest rate due to the endogenous response of monetary policy shock. To evaluate the net effect of these spillovers on the French economy we compare in the next subsection the multiplier in symmetric shock in the whole euro area with the one of an asymmetric shock located only in France.

Finally, comparing response of French variables in EA-BDF with those of FR-BDF, we can appreciate the size of spillovers from REA on the French economy. Two types of spillovers coexist. On the one hand, we have positive spillovers from trade due to higher foreign demand from the REA, which dampens the negative impact of exchange rate on real exports. And on the other hand, there are negative spillovers from higher nominal interest rate due to the endogenous response of monetary policy shock. As a result, aggregate spillovers appear limited but slightly positive on real GDP, in the very short run.[35]

5.2 Asymmetric shock and spillovers

In this case we shock only French government consumption while in the rest of the euro area there is no increase in government spending. Otherwise, we follow the same implementation as in the previous section. The results are presented in Figure 5.2.1 which also contains the results of the symmetric case for France to ease comparison.

In the asymmetric case on average during the first 2 years the multipliers are equal to 1 under VBE and 0.9 under MCE.[36] Such multipliers are close to those obtained in the symmetric case (1.1 under VBE and 1.0 under MCE). Even though the response of the interest rate is three times smaller in the asymmetric case, the fact that the net trade falls more during the shock explains the similar reaction of the French economy to the stimulus within this period. This leads us to the conclusion that the trade spillovers compensate in the short run spillovers related to monetary policy and it holds for both expectation types.

We observe some differences between MCE and VBE cases. As after a symmetric government spending shock, we observe less crowding-in of household consumption in the MCE case than under VBE because of the fall of permanent income related to the future transfer-based consolidation (reinforced by weaker tax receipts due to the smaller multiplier). There is also less crowding-in of private spending than under VBE because of the expected real rate increase generated by expected fall of the FR/REA relative price, as in Nakamura & Steinsson (2014).

Finally, the propagation mechanism is the same as in the symmetric case. We observe a crowding-in of household consumption and investment related to non-Ricardian features and non-aggressiveness of transfer rule. At the same time, there is a crowding-out of net trade caused at first by import content of public demand and then by competitiveness loss. The latter ensures in

[35] A more persistent government spending shock (e.g. +1% of GDP during 2 years) would trigger larger spillovers from trade.

[36] In the 10-year horizon the results look different: multiplier in the MCE case is higher than in the VBE, 0.16 and 0.13 respectively. This is due to smaller downswing in the MCE case.

the short run slightly positive spillovers for the rest of the euro area from fiscal stimulus in France, which are not fully counteracted by the monetary policy response, see Figure A.5.

The same results are found in Alloza et al. (2020) with a multi-country DSGE model: the spillovers by origin, i.e. the overall effect of government spending in one country of the union on others, is positive for all countries including France.[37] As reported in the paper, when there is a one percentage increase in government spending in France, the two-year average spillover on REA in terms of percentage change in GDP in this case is around 0.05 for France. We find a number of similar magnitude around 0.02.[38] Our results however differ from theirs for the spillovers by destination, i.e. the overall effect of government spending in all the union but one country on the latter. In the case of the government consumption shock, the negative effects of the monetary policy increase prevails positive trade effects in Alloza et al. (2020). For comparison, we run an additional experiment where fiscal consolidations increase only in REA countries and not in France. As in the case of a shock originating in France, we also find small but positive spillovers in this case, around 0.10 on average during the first two years (see Figure A.6).

5.3 Interaction with different monetary policy rules

In this section, we address the question of fiscal policy *effectiveness* and its sensitivity to different monetary policy rules and alternative expectation formation mechanisms. In particular, we evaluate how sensitive are fiscal multipliers to different type of monetary policy rules and to alternative expectation formation mechanisms.[39]

Simulations' protocol First, we consider both model-consistent expectations and hybrid expectations versions and let aside the VAR-based expectations version of EA-BDF.[40] The case with hybrid expectations correspond to a situation of heterogenous types of expectations depending on agent types: in such a case, financial agents have model-consistent expectations, while non-financial agents have VAR-based expectations.

As in section 5.1, we simulate a symmetric shock to France and REA government real consumption of +1% of long-run GDP, kept constant during 2 years and gradually reduced afterward with a persistence $\rho_G = 0.8$.

We compare results under three alternative monetary policy rules. We choose the same specifications and calibrations of these rules as in Pedersen et al. (2021), in order to ease the comparison

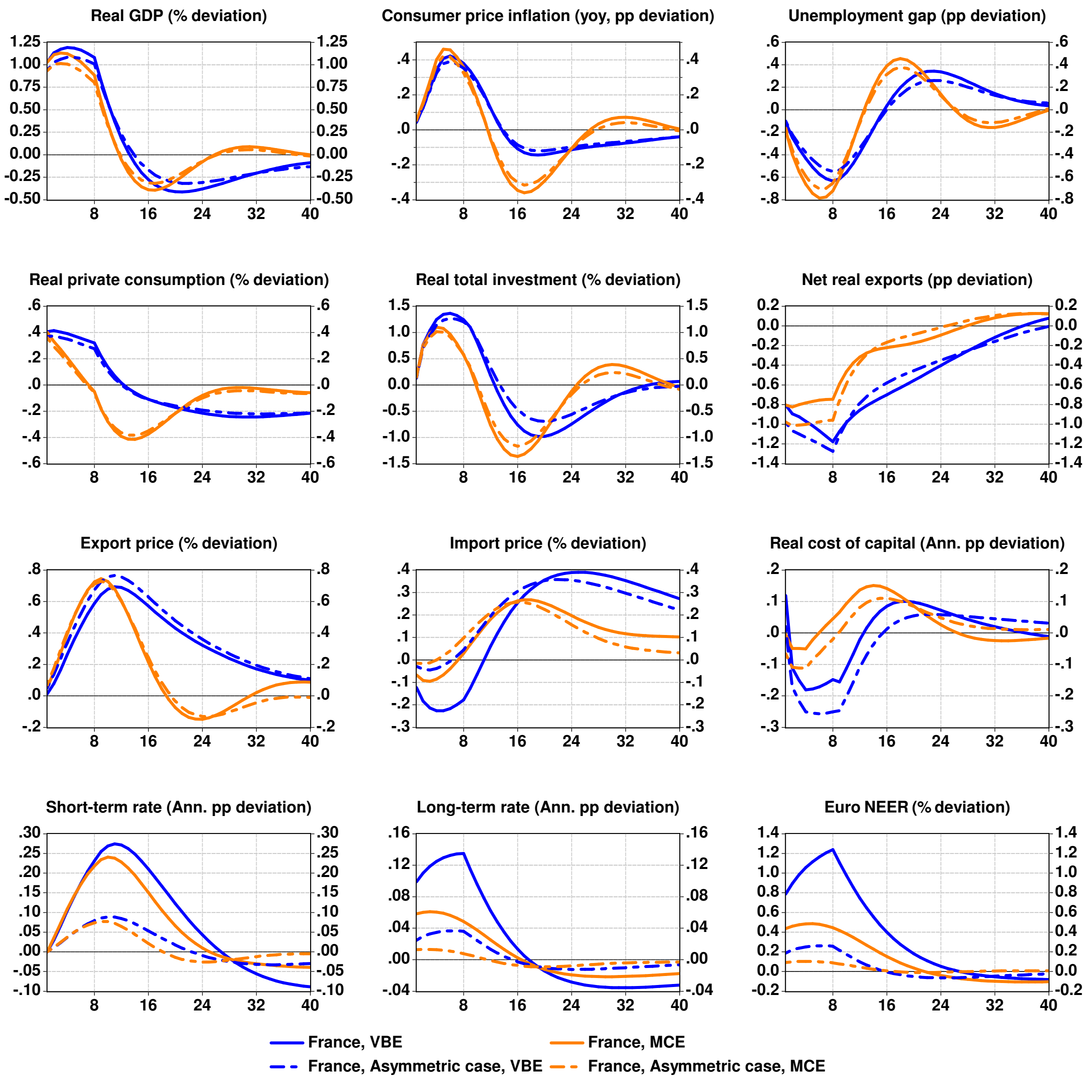
[37] We focus here only on their simulations related to government consumption, which are comparable to ours.

[38] As in Alloza et al. (2020) they use DSGE model for analysis, we report our results for the MCE case here.

[39] We do a similar application in our contribution to Pedersen et al. (2021), the main differences being that we use here a more recent version of EA-BDF and that we let here the fiscal rule activated during all simulations.

[40] The VBE case is not relevant in this exercise. First, because monetary policy behaves similarly under each type of rule in the short-run, differences materialize only in the medium-run. Then, these medium-run differences does not affect agents' expectations in the short run because they are backward looking. As a result, responses of the VBE version of the model do not depend much monetary policy rules.

Figure 5.2.1: Government consumption shock, IRFs for France, symmetric and asymmetric cases



of our results with theirs. First, we consider an Inflation Targeting (IT) monetary policy rule:

$$i^4_{EA,t} = \rho i^4_{EA,t-1} + (1-\rho)\left[\bar{r}^4_{EA} + \pi^4_{EA,C,t} + \hat{y}_{EA,t} + 0.5(\pi^4_{EA,C,t} - \bar{\pi}^4_{EA})\right] + \varepsilon_t \tag{57}$$

where $i^4_{EA,t}$ is the annualized nominal interest rate, $\bar{r}^4_{EA}$ is the annual real interest rate at steady-state, $\pi^4_{EA,C,t}$ is the *year-on-year* consumer price inflation. $\bar{\pi}^4_{EA}$ is the steady-state annual inflation rate. The second rule we consider is a Price-Level Targeting (PT) monetary policy rule:

$$i^4_{EA,t} = \rho i^4_{EA,t-1} + (1-\rho)\left[\bar{r}^4_{EA} + \pi^4_{EA,C,t} + \hat{y}_{EA,t} + \hat{p}_{EA,C,t}\right] + \varepsilon_t \tag{58}$$

where $\hat{p}_{EA,C,t} \equiv \log(P_{EA,C,t}/\bar{P}_{EA,C,t})$ is the log-deviation of the consumption price from its trend

$\bar{P}_{EA,C,t}$, which is assumed to grow at rate $\bar{\pi}_{EA}$. Finally, we consider an Average-Inflation Targeting (AIT) rule over 4 years:

$$i^4_{EA,t} = \rho i^4_{EA,t-1} + (1-\rho)\big[\bar{r}^4_{EA} + \pi^4_{EA,C,t} + \hat{y}_{EA,t} + N\big(\frac{1}{N}\sum_{i=1}^{4N} \pi_{EA,C,t-i+1} - \bar{\pi}^4_{EA}\big)\big] + \varepsilon_t \quad (59)$$

where $N = 4$ and $\pi_{EA,C,t}$ is the *quarterly* rate of consumer price inflation. Persistence (or policy inertia) is homogenous across monetary policy rules, with $\rho = 0.85$.

Finally, we focus on two alternative scenarios. In the "Baseline" scenario, monetary policy is always active and endogenously reacts to the shock. In the "Fix rate" scenario, we assume that monetary policy stays passive and keeps its policy rate unchanged during the shock (8 quarters) and becomes active afterward. Given that we run simulations around the balanced-growth path, the initial price gap $\hat{p}_{EA,C,t}$ is equal to zero and this "Fix rate" scenario proxies a zero-lower-bound scenario but without any backlog of the price level. In both scenarios, we keep the fiscal policy rule activated during the shock, such that government adjusts fiscal transfers to stabilize the nominal deficit ratio toward the debt-stabilizing deficit-to-GPD ratio.[41]

Results Figure 5.3.1 shows the first and second year fiscal multipliers under each monetary policy rule and each expectation type, for the "Baseline" and "Fix rate" scenarios. Figures 5.3.2 and A.7 in appendix provides selected impulse responses functions to further understand similarities and differences between the different simulations.

First, when monetary policy is active, we find quite similar fiscal multipliers whatever the type of monetary policy rules and the type of expectations. Second, when monetary policy is passive, fiscal multipliers are always larger than in normal times, whatever the expectation types. We also find that the PT monetary policy rule yields lower fiscal multipliers, due to a smaller impact of government consumption on inflation expectations (due to the future reversal of the price level) but also to additional monetary policy tightening compare to an inflation targeting rule, see Figures 5.3.2 for the MCE case[42]. On the contrary, the latter displays the largest fiscal policy multipliers. Third, average-inflation targeting rules always display intermediate results, which are generally closer to those obtained with a price-level targeting rule. Finally, at least in the "Fix rate" scenario, we also find smaller multipliers under MCE compared to HYB, due to some Ricardian effects that are absent in the HYB case, because households are not aware of future fiscal adjustment in the latter.

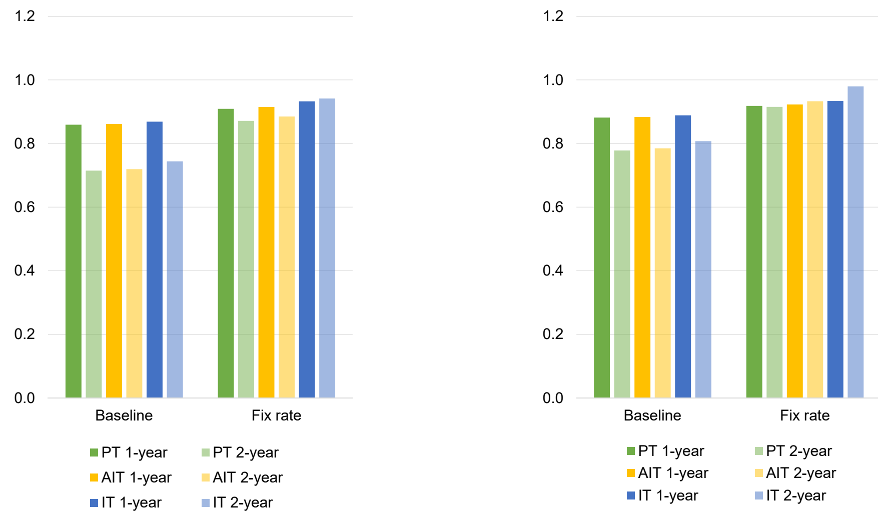
Main results are robust to the type of expectations, although fiscal multipliers are slightly larger and more persistent under hybrid expectations compared to model-consistent expectations. The latter can be explained by the smaller Ricardian effects and consequently the larger response of private consumption compared to the MCE case.

Our results are very intuitive and mostly in line with those obtained with DSGE models of the

[41] As already mentioned in section 2.10, our deficit-stabilization rule is relatively non-aggressive. As a consequence, if we deactivated the fiscal rule during the period of the shock, results would be qualitatively unchanged.

[42] It is also visible in the hybrid case, see A.7 in appendix

Figure 5.3.1: Government consumption multipliers in the euro area under different monetary policy rules and expectation formation mechanisms



(a) Model-consistent expectations

(b) Hybrid expectations

euro area in Pedersen et al. (2021), although some differences are notable. First, fiscal multipliers are found less persistent with these DSGEs compared to our semi-structural approach. This larger persistence might be related to larger real and nominal rigidities of our semi-structural model with polynomial adjustment costs than in these DSGE models. Second, difference between fiscal multipliers across monetary regimes ("Baseline" and "Fix rate") are smaller with our semi-structural model than with these DSGEs. This might be due to the smaller interest-rate sensitivity of household consumption in our semi-structural model compared to DSGEs. The latter dampens the effects of monetary policy tightening following the shock in normal times ("Baseline" scenario) but also dampens the effects of inflation expectations generated by the government spending stimulus when monetary policy is passive ("Fix rate" scenario).

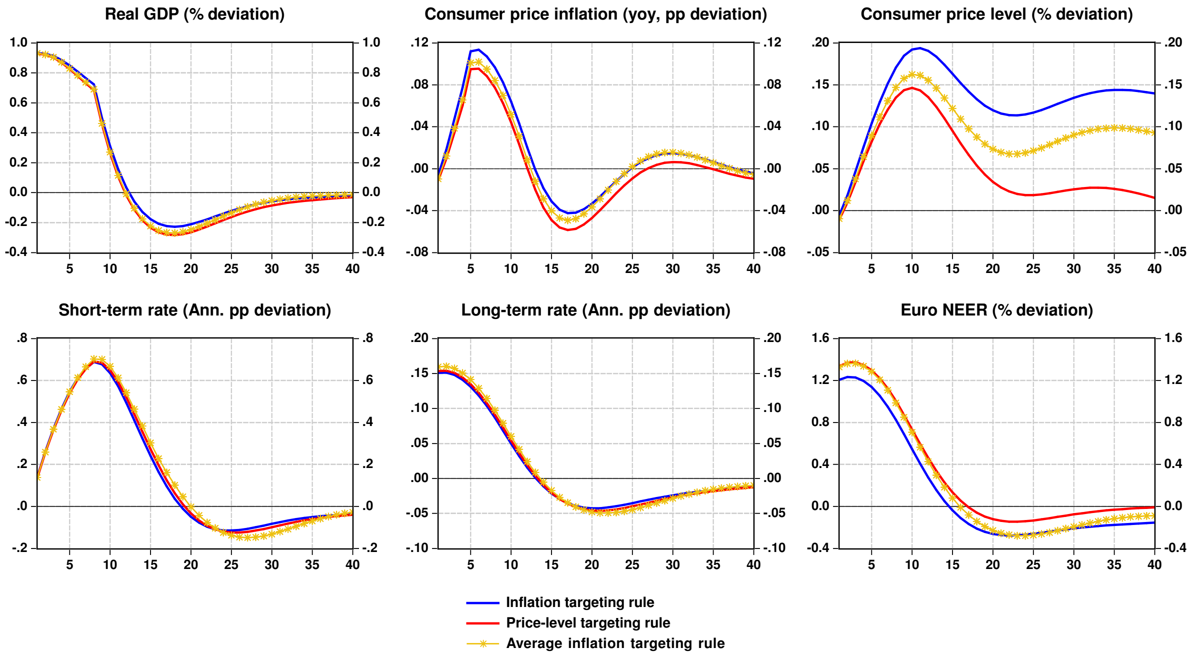
6 Conclusion

In conclusion, our new two-country model of the euro area (EA-BDF) has four nice features. First, although our REA block is less detailed than our French block based on FR-BDF, we have some consistency between general approaches of both blocks. Second, we can run simulations of this model under different types of expectations. Third, thanks to our multi-country setup, we can deal with both symmetric shocks in the whole euro area or with asymmetric shocks origination in France or REA. Fourth, it allows for endogenous monetary policy and, hence, for studying stabilization properties of alternative monetary policy rules.

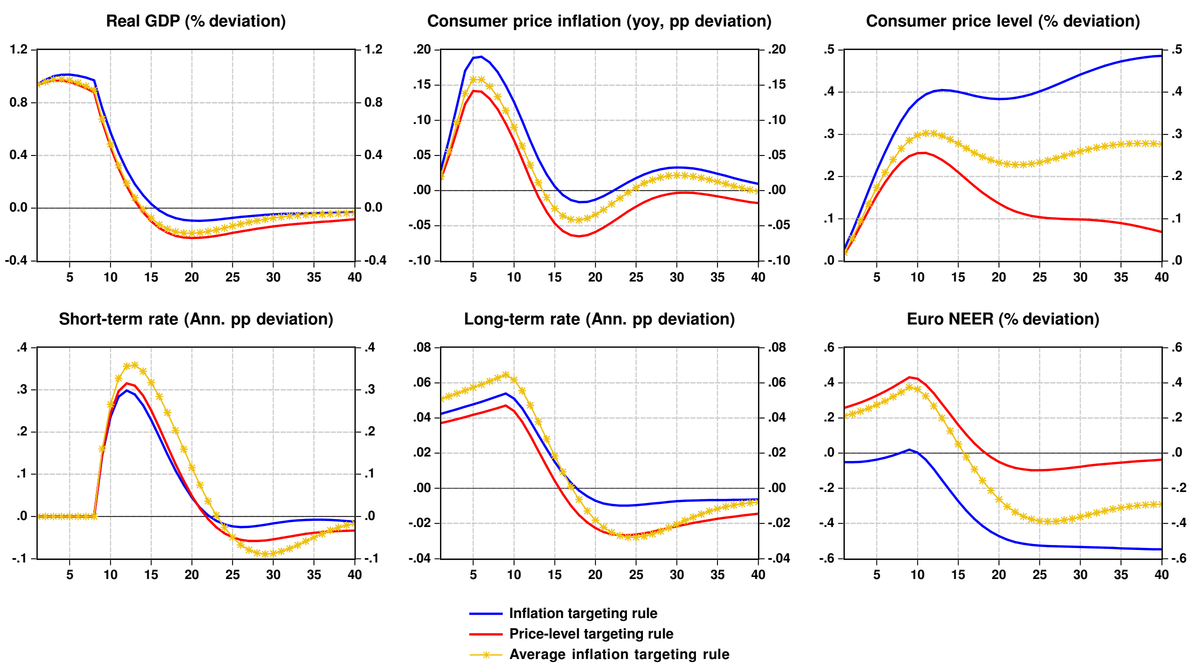
Our illustrations about the effects of a government spending shock in a monetary union deliver in particular two interesting results. First, by studying symmetric and asymmetric shocks on government spending, kept constant for 2 years, we find that, at this 2-year horizon, trade spillovers would slightly dominate monetary policy spillovers within the euro area. Second, we also find, in the case of a symmetric shock, that the government spending multiplier is smaller under a monetary policy rule based on price-level targeting than on inflation targeting.

In further research, the next step regarding the study of the effects of fiscal policy would be to study the effects of shocks on government investment, which play a key role in current recovery plans like for example the "Next Generation European Union" fiscal package of the European Commission. Concerning the effects of monetary policy, within the current long-lasting liquidity trap, non-standard measures like asset purchases and forward guidance have today a prominent role and it would be interesting to study the impact of such policies in this new modeling framework.

Figure 5.3.2: Effects of a government consumption shock in the euro area under different monetary policy rules and model-consistent expectations



(a) Baseline: Active monetary policy



(b) Fix rate: Passive monetary policy during 8 quarters

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A Appendix

A.1 Data construction for external trade of the rest of euro area

In this appendix, we detail how we construct time series of the rest of euro area (REA) used in equations related to external trade. First, we explain how we decompose gross trade flows (imports and exports) of this area between consolidated and internal flows. Second, we explain how we compute some usual trade consistency indicators (foreign demand and competitor prices).

In order to decompose gross trade flows of REA into consolidated and internal flows, we rely on data available for the euro area and for France with respect to intra/extra decompositions. For internal imports of REA ($p_{M,REA,INT,t}M_{REA,INT,t}$), we substract to intra imports of the euro area ($p_{M,EA,IN,t}M_{EA,IN,t}$) imports external to the rest of euro area, i.e. imports of France from REA (intra imports of France, denoted $p_{M,FR,IN,t}M_{FR,IN,t}$) and imports of REA from France (proxied by intra exports of France, denoted $p_{X,FR,IN,t}X_{FR,IN,t}$).[A.1] For consolidated imports of REA ($p_{M,REA,CON,t}M_{REA,CON,t}$), we just substract to gross imports of REA ($p_{M,REA,t}M_{REA,t}$) internal imports of REA ($p_{M,REA,INT,t}M_{REA,INT,t}$). For internal and consolidated exports of REA ($p_{X,REA,INT,t}X_{REA,INT,t}$ and $p_{X,REA,CON,t}X_{REA,CON,t}$), we proceed in a symmetric way. We can summarize these computations by the following formulas:

$$
\begin{aligned}
p_{M,REA,INT,t}M_{REA,INT,t} &= p_{M,EA,IN,t}M_{EA,IN,t} - p_{M,FR,IN,t}M_{FR,IN,t} - p_{X,FR,IN,t}X_{FR,IN,t}, \\
p_{M,REA,CON,t}M_{REA,CON,t} &= p_{M,REA,t}M_{REA,t} - p_{M,REA,INT,t}M_{REA,INT,t}, \\
p_{X,REA,INT,t}X_{REA,INT,t} &= p_{X,EA,IN,t}X_{EA,IN,t} - p_{X,FR,IN,t}X_{FR,IN,t} - p_{M,FR,IN,t}M_{FR,IN,t}, \\
p_{X,REA,CON,t}X_{REA,CON,t} &= p_{X,REA,t}X_{REA,t} - p_{X,REA,INT,t}X_{REA,INT,t}.
\end{aligned}
$$

In order to compute trade consistency indicators of REA, we rely on data available for the euro area and France with respect to such indicators.[A.2] For foreign demand of REA ($d_{W,REA,t}$), we compute it as a geometric average of intra and extra foreign demands. The computation of the intra foreign demand ($d_{W,REA,IN,t}$) is simply based on French imports. As the extra foreign demand of euro area is a geometric average of the extra foreign demand adressed to French and REA exports, we can determine the extra foreign demand of REA ($d_{W,REA,EX,t}$) based on those of EA and France. In the end, we can summarize these computations with the following formulas:

$$
\begin{aligned}
d_{W,REA,IN,t} &= m_{FR,t} \\
d_{W,REA,EX,t} &= \frac{d_{W,EA,t} - \phi_{DW,FR,t} d_{W,FR,EX,t}}{1 - \phi_{DW,FR,t}} \\
d_{W,REA,t} &= \omega_{DW,IN,t} d_{W,REA,IN,t} + (1 - \omega_{DW,IN,t})\, d_{W,REA,EX,t}
\end{aligned}
$$

[A.1] We provide here formulas for values. We use analogue formulas for volumes with chained-price aggregation techniques.

[A.2] For deriving formulas provided hereafter, we start from definitions of foreign demand and competitor prices provided in Hubrich & Karlsson (2010).

where $\phi_{DW,FR}$ is the weight of French extra exports in EA consolidated exports and $\omega_{DW,IN}$. is the weight of REA exports toward France (proxied by French intra imports) in consolidated exports of REA (computed above). Both weights are computed as moving average of ratios over three years.

For competitor prices of REA on the import side ($p_{CM,REA,t}$) and its intra/extra components ($p_{CM,REA,IN,t}$ and $p_{CM,REA,EX,t}$), we proceed in a similar way based on the following formulas:

$$
\begin{aligned}
p_{CM,REA,IN,t} &= p_{X,FR,t} \\
p_{CM,REA,EX,t} &= \frac{p_{CM,EA,t} - \phi_{PCM,FR,t} p_{CM,FR,EX,t}}{1 - \phi_{PCM,FR,t}} \\
p_{CM,REA,t} &= \omega_{PCM,IN,t} p_{CM,REA,IN,t} + (1 - \omega_{PCM,IN,t})\, p_{CM,REA,EX,t}
\end{aligned}
$$

where $\phi_{DW,FR}$ is the weight of French extra imports in EA consolidated imports and $\omega_{DW,IN}$. is the weight of REA imports from France (proxied by French intra exports) in consolidated imports of REA (computed above). Both weights are computed as moving average of ratios over three years.

For competitor prices of REA on the export side ($p_{CX,REA,t}$) and its intra/extra components ($p_{CX,REA,IN,t}$ and $p_{CX,REA,EX,t}$), because of double-weighting schemes related to third-party effects, it appears difficult to find formulas relating theim to corresponding indicators for EA and France. Hence, for simplifying purposes, we proxy them with corresponding indicators on the import side.

A.2 Marshall-Lerner condition

In order to assess whether an exchange rate depreciation improves the trade balance we derive the Marshall-Lerner condition. We derive this condition in an incomplete pass-through setting (and in the long run) following Bussière et al. (2020). First we define the trade balance (TB):

$$
TB = P_{REA,X,CON,t} X_{REA,CON,t} - P_{REA,M,CON,t} M_{REA,CON,t}
$$

where both export and import prices are here expressed in the domestic currency. Next, we take the derivative of the trade balance with respect to the nominal effective exchange rate E (defined such as $dE > 0$ is an appreciation).

$$
\begin{aligned}
\frac{dTB}{dE} &= X_{REA,CON,t} \frac{dP_{REA,X,CON,t}}{dE} + P_{REA,X,CON,t} \frac{dX_{REA,CON,t}}{dE} \\
&\quad - \left(M_{REA,CON,t} \frac{dP_{REA,M,CON,t}}{dE} + P_{REA,M,CON,t} \frac{dM_{REA,CON,t}}{dE} \right) \qquad \text{(A.1)}
\end{aligned}
$$

As X is a function of $P_{REA,X,CON,t} \times E$ and not only of $P_{REA,X,CON,t}$, its derivative with respect to E verifies:

$$
\begin{aligned}
\frac{dX_{REA,CON,t}}{dE} &= \frac{1}{E} \frac{d(P_{REA,X,CON,t} E)}{dE} \frac{dX_{REA,CON,t}}{dP_{REA,X,CON,t}} \\
&= \left(\frac{dP_{REA,X,CON,t}}{dE} + \frac{P_{REA,X,CON,t}}{E} \right) \frac{dX_{REA,CON,t}}{dP_{REA,X,CON,t}} \qquad \text{(A.2)}
\end{aligned}
$$

Then, we get from A.1 and A.2:

$$\begin{aligned}\frac{dTB}{dE} &= \frac{dP_{REA,X,CON,t}}{dE}X_{REA,CON,t} + \\ & P_{REA,X,CON,t}\frac{dX_{REA,CON,t}}{dP_{REA,X,CON,t}}\left(\frac{dP_{REA,X,CON,t}}{dE}+\frac{P_{REA,X,CON,t}}{E}\right) - \\ & \left(M_{REA,CON,t}\frac{dP_{REA,M,CON,t}}{dE}+P_{REA,M,CON,t}\frac{dM_{REA,CON,t}}{dP_{REA,M,CON,t}}\frac{dP_{REA,M,CON,t}}{dE}\right)\end{aligned}$$

And

$$\begin{aligned}\frac{dTB}{dE}\frac{E}{TB} &= -\frac{P_{REA,X,CON,t}X_{REA,CON,t}}{TB}\left(\beta_{P_X}+\beta_X(1-\beta_{P_X})\right) + \\ & \frac{P_{REA,M,CON,t}M_{REA,CON,t}}{TB}\left(\beta_{P_M}-\beta_M\beta_{P_M}\right)\end{aligned}$$

Where

$$\begin{aligned}\beta_{P_X} &= -\frac{dP_{REA,X,CON,t}}{dE}\frac{E}{P_{REA,X,CON,t}} \\ \beta_{P_M} &= -\frac{dP_{REA,M,CON,t}}{dE}\frac{E}{P_{REA,M,CON,t}}\end{aligned}$$

are the (absolute values of) exchange rate pass-through elasticities of trade prices, and

$$\begin{aligned}\beta_X &= -\frac{dX_{REA,CON,t}}{dP_{REA,X,CON,t}}\frac{P_{REA,X,CON,t}}{X_{REA,CON,t}} \\ \beta_M &= -\frac{dM_{REA,CON,t}}{dP_{REA,M,CON,t}}\frac{P_{REA,M,CON,t}}{M_{REA,CON,t}}\end{aligned}$$

are the (absolute value of) price elasticities of respectively export and import volumes.

In practice, $\beta_X > 0$ and $\beta_M > 0$ because a higher export (resp. import) prices depress export (resp. import) volumes. Both β_{P_X} and β_{P_M} lie in $[0, 1]$:

- For β_{P_X}, an increase in E (ie an appreciation) yields a decrease in $P_{REA,X,CON,t}$ denominated in euro as exporters try to dampen their competitiveness loss in foreign markets. Symmetrically, firms can rise their margins by increasing their prices in euros when they benefit from a depreciated exchange rate. They do not fully pass-through exchange rate into their prices expressed in the foreign currency. Full pass-through is $\beta_{P_X} = 0$ and zero pass-through is $\beta_{P_X} = 1$ (constant prices of exports in foreign currency), leading to $0 \geq \beta_{P_X} \geq 1$.

- For β_{P_M}, an increase in E yields a decrease in the import price denominated in euro ($P_{REA,M,CON,t}$). On one hand, full pass-through is $\beta_{P_M} = 1$. As foreign firms pass-through by less than one-to-one exchange rate into their prices in euro (they increase their markups) β_{P_M} can be inferior to 1. On the other hand $\beta_{P_M} = 0$ when there is no pass-through (pricing to market by foreign firms). Full pass-through is $\beta_{P_M} = 0$ and zero pass-through is $\beta_{P_M} = 1$, leading to $0 \geq \beta_{P_M} \geq 1$.

Under the assumption that trade is balanced

$$P_{REA,X,CON,t} X_{REA,CON,t} = P_{REA,M,CON,t} M_{REA,CON,t}$$

the Marshall-Lerner condition that the trade balance improves after an exchange rate depreciation becomes:

$$\begin{aligned} -(\beta_{P_X} + \beta_X(1-\beta_{P_X})) + (\beta_{P_M} - \beta_M \beta_{P_M}) &< 0 \\ \beta_X + \beta_{P_X}(1-\beta_X) - \beta_{P_M}(1-\beta_M) &> 0 \end{aligned} \tag{A.3}$$

Notice that under full pass-through ($\beta_{P_X} = 0$ and $\beta_{P_M} = 1$) this condition becomes $\beta_X + \beta_M > 1$. Notice too that the condition that we get is equivalent to the one found in Bussière et al. (2016). First, we can relate our elasticities with theirs, denoted here β_X^{BGS} and μ_X^{BGS}, in the following way:

$$\begin{aligned} \beta_X^{BGS} &= -\frac{\frac{d(EP_{REA,X,CON,t})}{d(\frac{1}{E})}}{\frac{EP_{REA,X,CON,t}}{\frac{1}{E}}} \\ &= (1-\beta_{P_X}) \end{aligned} \tag{A.4}$$

$$\begin{aligned} \mu_X^{BGS} &= -\frac{\frac{dX_{REA,CON,t}}{d(EP_{REA,X,CON,t})}}{\frac{X_{REA,CON,t}}{EP_{REA,X,CON,t}}} \\ &= \beta_X \end{aligned} \tag{A.5}$$

Then, we can recover from equations A.3, A.4 and A.5 the BGS condition as follows:

$$\begin{aligned} \mu_X^{BGS} + (1-\beta_X^{BGS})(1-\mu_X^{BGS}) - \beta_X^{BGS}(1-\mu_X^{BGS}) > 0 \\ \beta_X^{BGS}(1-\mu_X^{BGS}) + \beta_X^{BGS}(1-\mu_X^{BGS}) < 1 \end{aligned}$$

If we calculate the effective Marshall-Lerner condition in our model, given the estimates of trade elasticities obtained in 2.7, i.e. $\beta_{P_X} = 0.14$, $\beta_{P_M} = 0.77$[A.3], $\beta_X = 0.9$, $\beta_M = 0.1.49$, we have

$$\beta_X + \beta_{P_X}(1-\beta_X) - \beta_{P_M}(1-\beta_M) = 1.29$$

So, the ML condition is easily met. But we still have to take into account the indirect effect of the exchange rate on export price via the import price, ie the import content of exports. In equation 31 the long run elasticity of the export price to the import price is 0.31 (β_1). The ML condition

[A.3] In the case of the elasticity of the import price to the exchange rate, β_{P_M}, we compute it from the elasticity of this price to the price of foreign exporters (0.84), weighted by the share of extra euro area (0.89).

becomes:

$$\beta_X + (\beta_{P_X} + 0.31\beta_{P_M})(1 - \beta_X) - \beta_{P_M}(1 - \beta_M) = 1.31$$

Finally, if the exchange rate variation is against the US dollar we must take into account the effect on import (and export) prices through the oil price. In equation 35, the long run elasticity of import price to the oil price is 0.095 (β_2). The augmented ML condition is:

$$\beta_X + (\beta_{P_X} + 0.31(\beta_{P_M} + 0.095))(1 - \beta_X) - (\beta_{P_M} + 0.095)(1 - \beta_M) = 1.36$$

A.3 Additional impulse responses functions

Figure A.1: Long-term interest rate shock (term-premium), IRFs for the REA

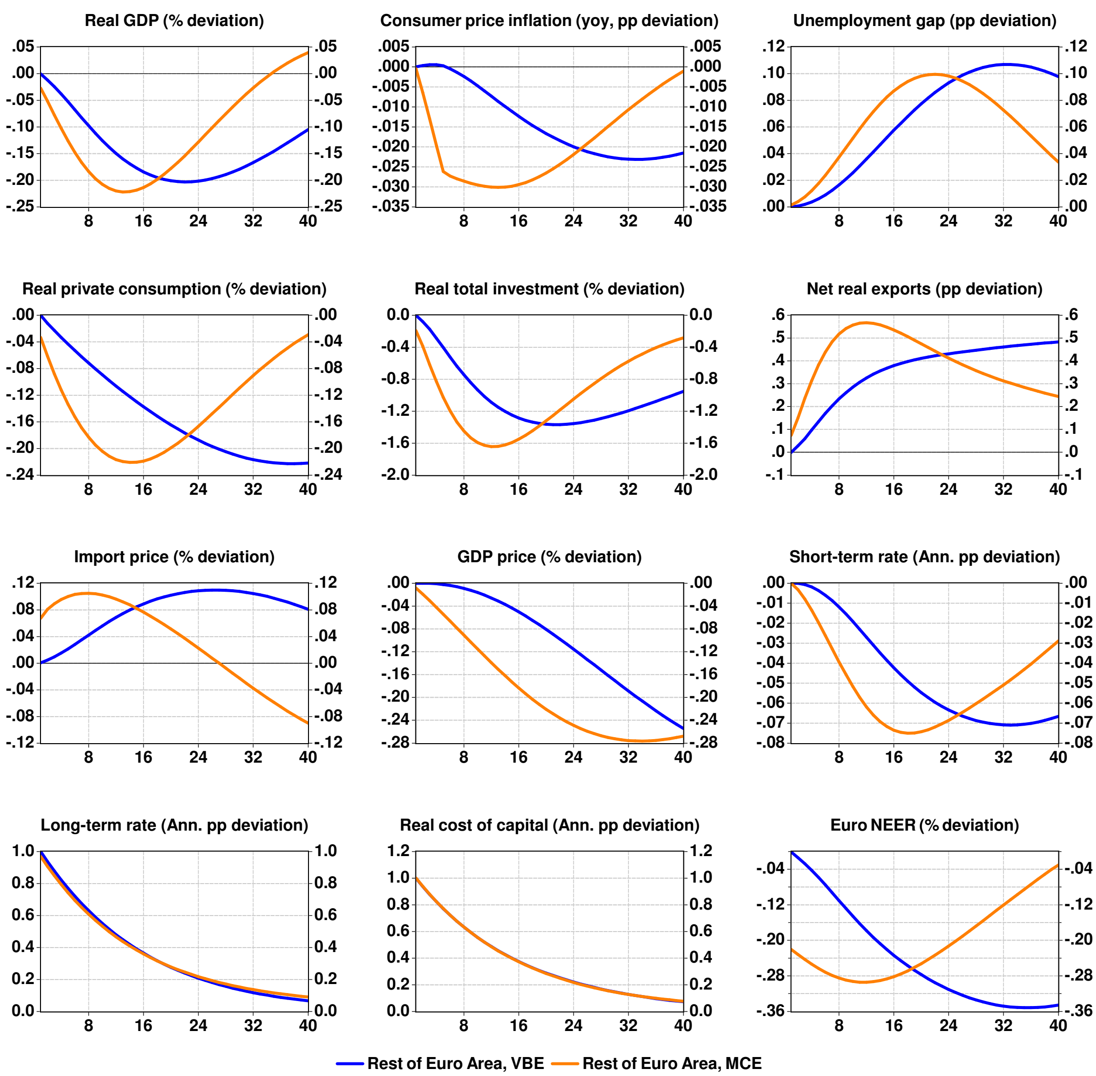


Figure A.2: Nominal exchange rate shock (risk-premium), IRFs for the REA

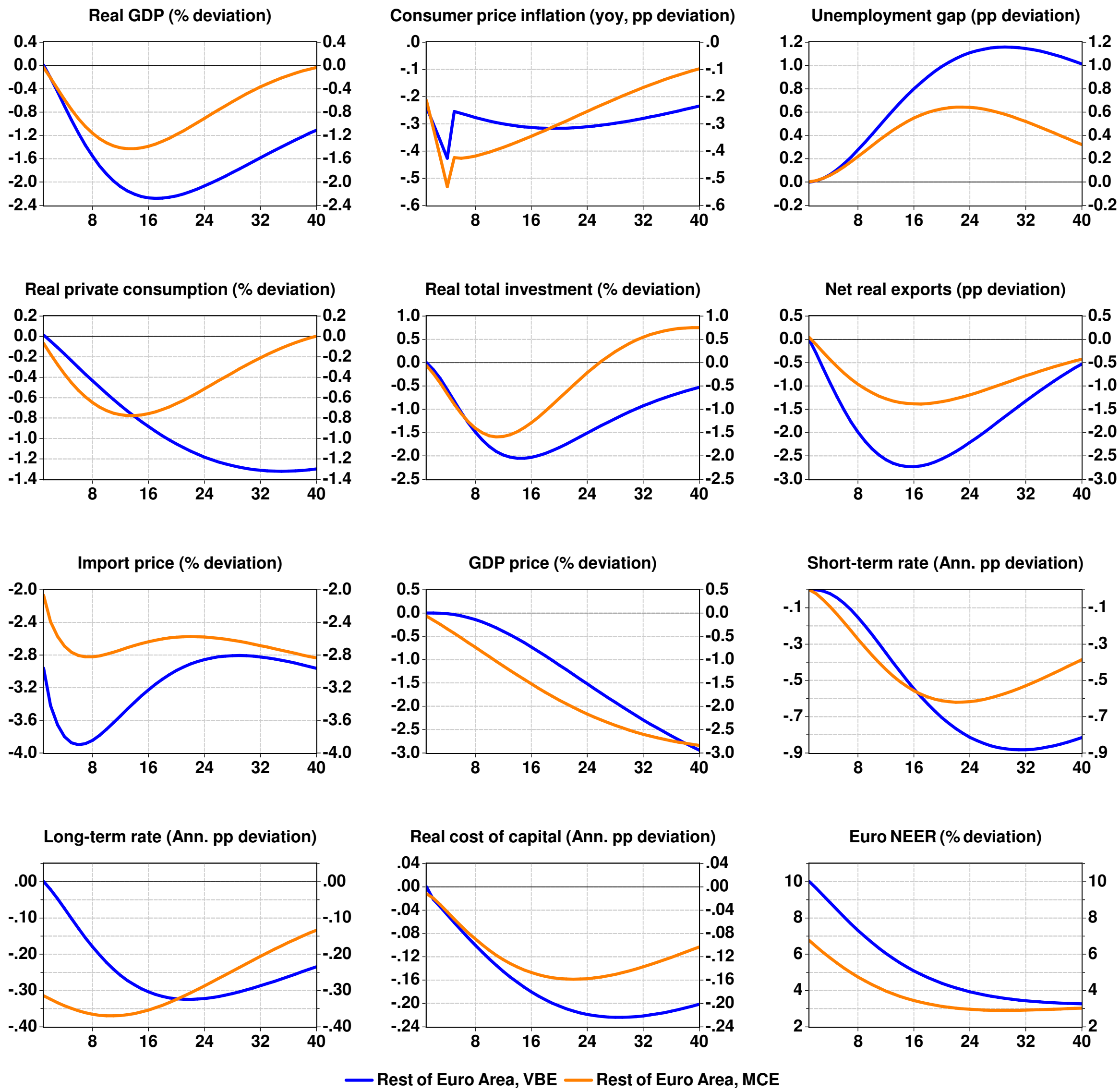


Figure A.3: Long-term interest rate shock (term-premium), IRFs for France

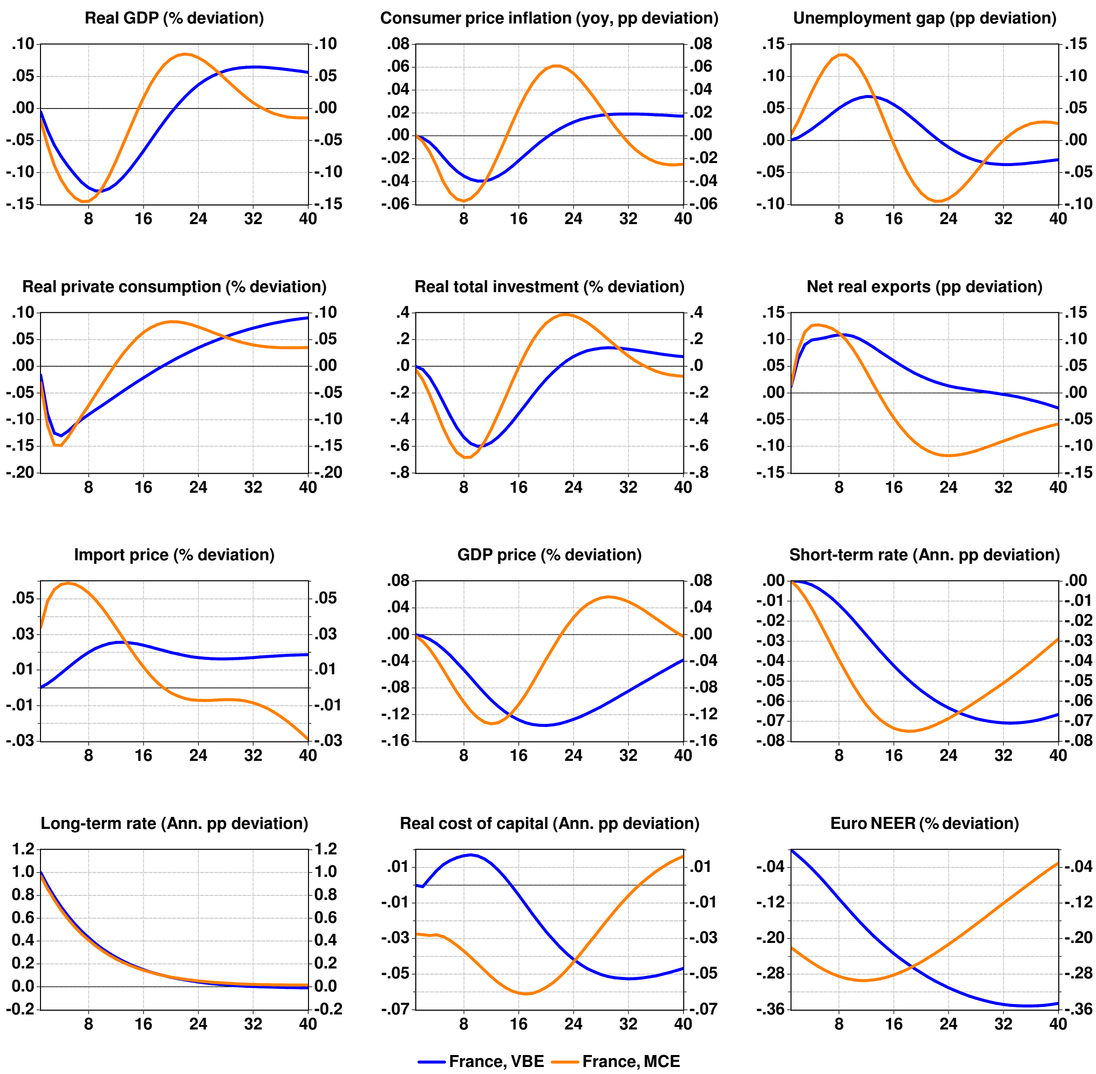


Figure A.4: Nominal exchange rate shock (risk-premium), IRFs for France

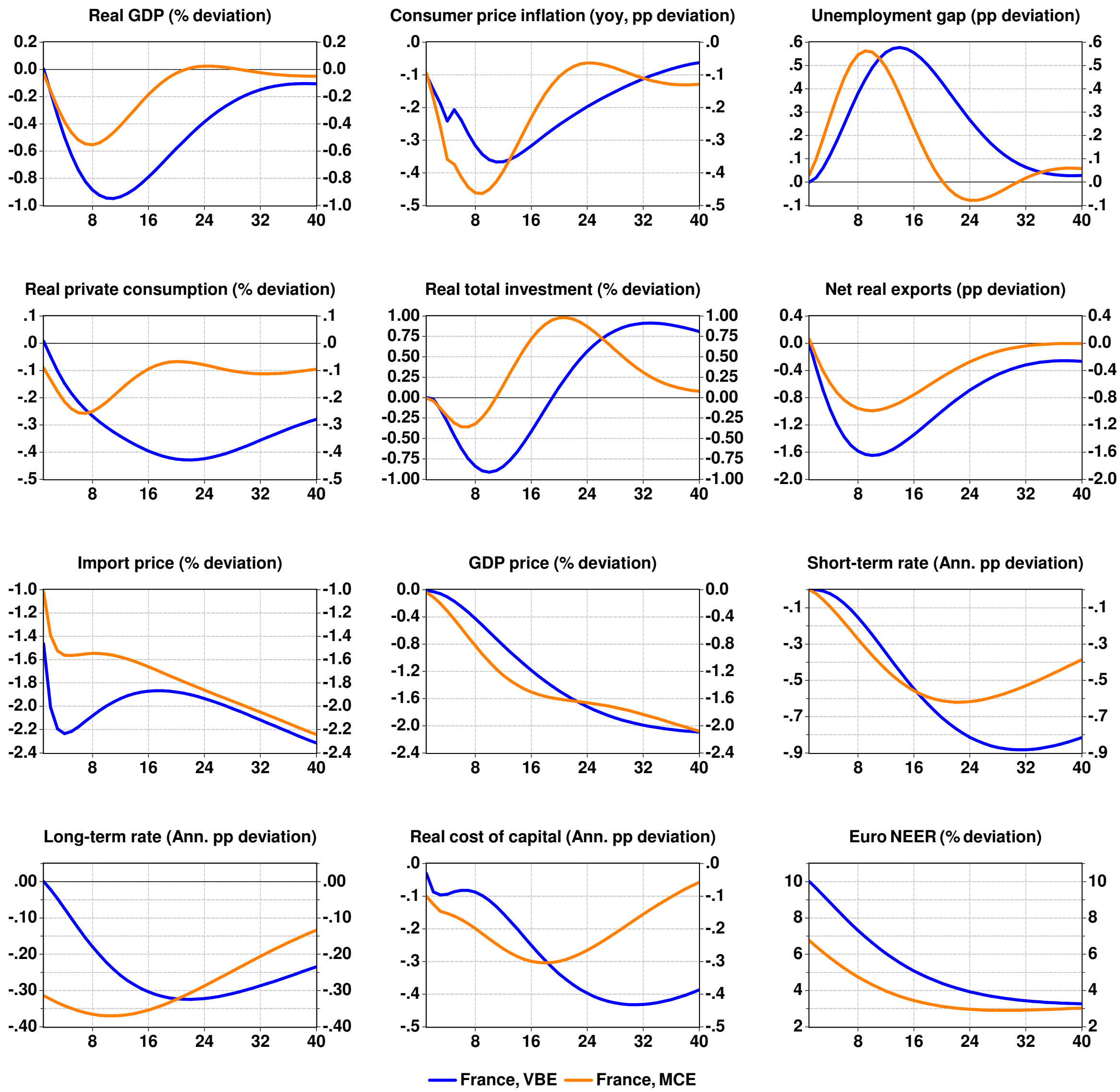


Figure A.5: Government consumption shock in France (asymmetric case), IRFs for the REA

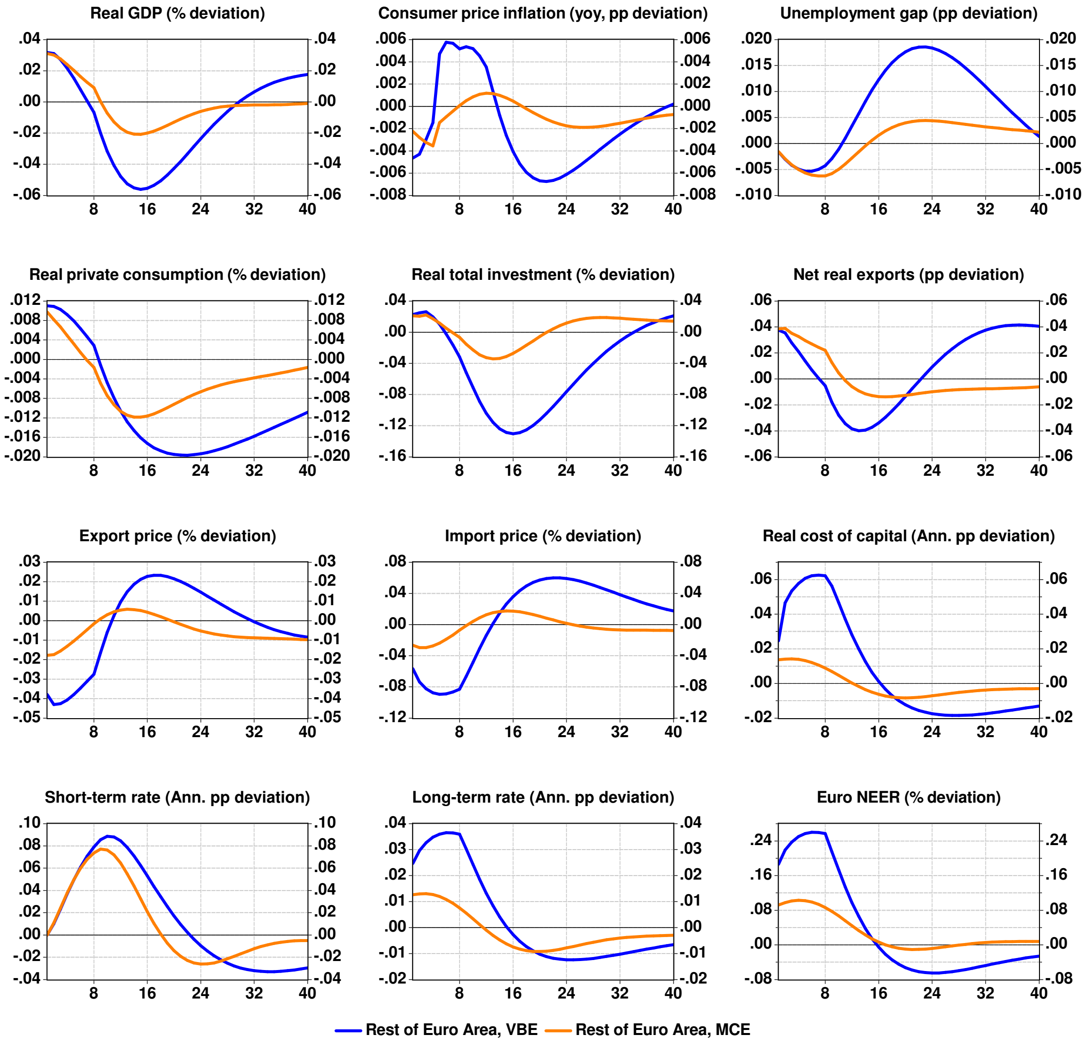
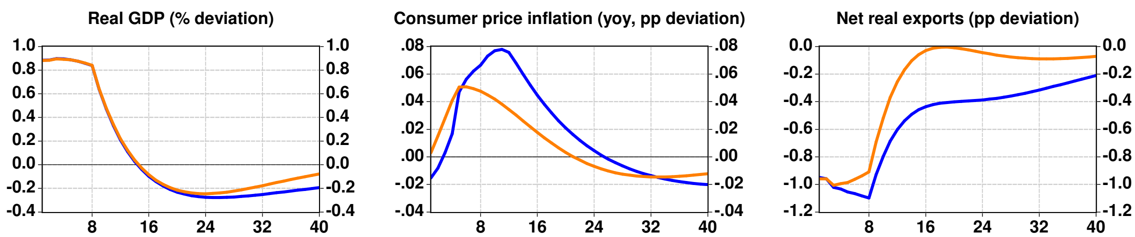
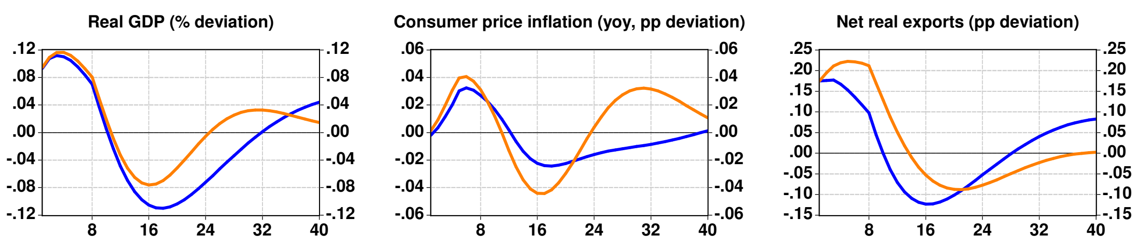


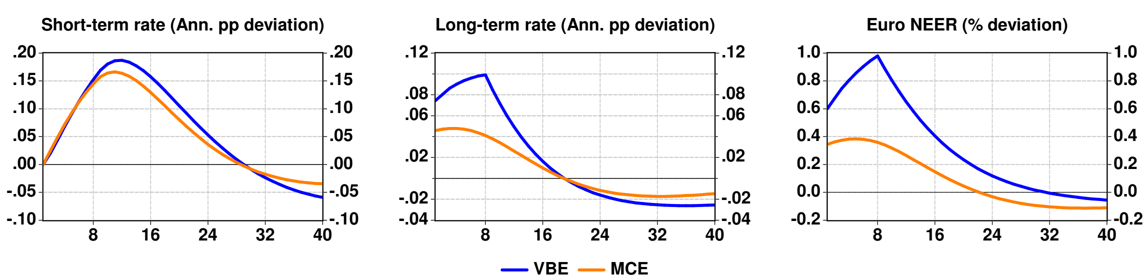
Figure A.6: Government consumption shock in all countries of EA but France (asymmetric case), IRFs for the REA and France



(a) REA responses

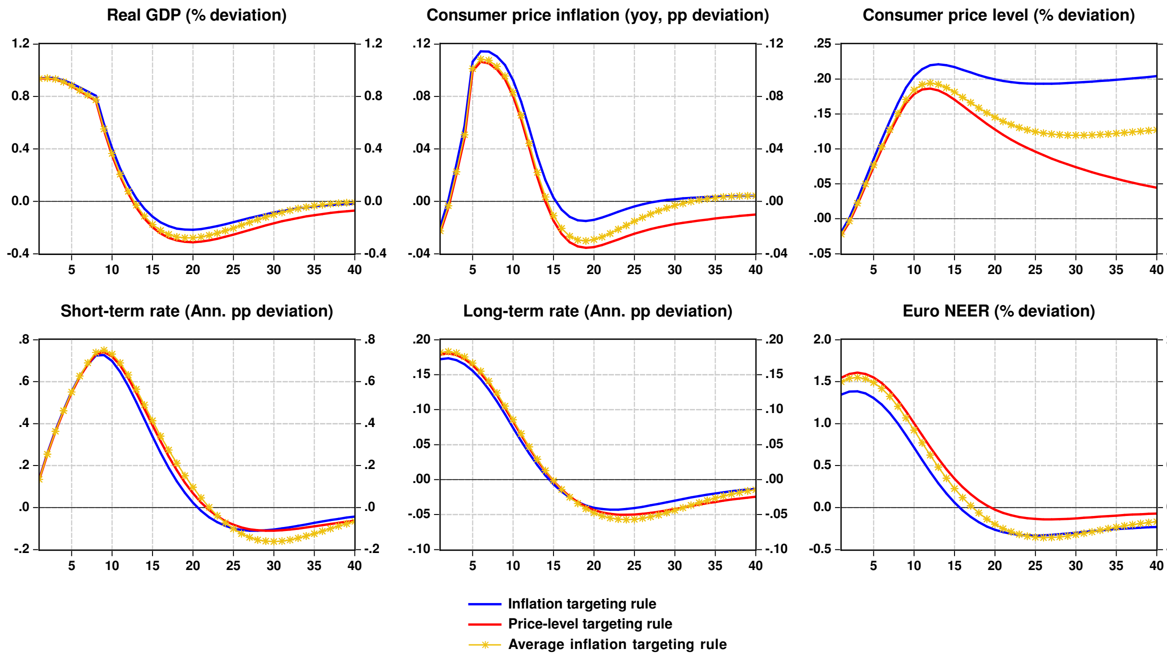


(b) French responses

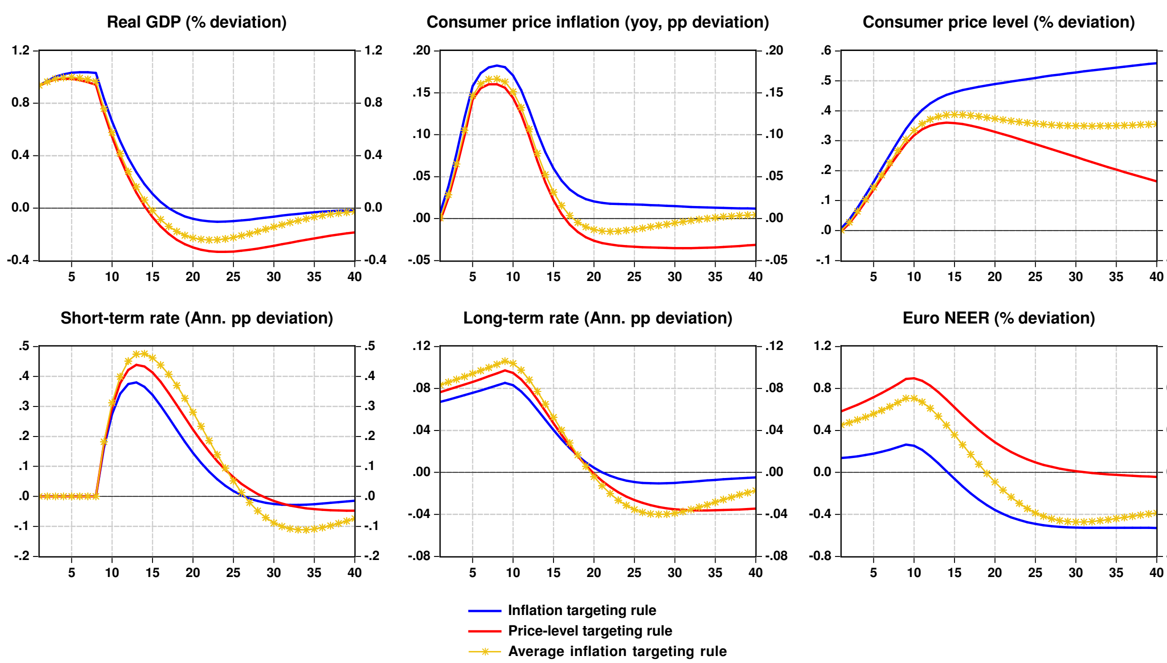


(c) Financial variables: short-run interest rate and exchange rate are those of EA, long-term rate response is identical for France and REA as the term premium is unchanged.

Figure A.7: Effects of a government consumption shock in the euro area under different monetary policy rules and hybrid expectations



(a) Baseline: Active monetary policy



(b) Fix rate: Passive monetary policy during 8 quarters