WORKING paper

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Privilege Lost? The Rise and Fall of a Dominant Global Currency

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Abstract

How does a country obtain the status of a safe haven with a dominant global currency? This paper argues that size matters: as a country becomes larger and more diversified, the underlying shock process of the economy becomes less variable. Shocks that can drive a government to default become less likely, implying lower default probability, lower interest rates and higher debt-to-GDP. Furthermore, the larger a country's share in the supply of global safe assets, the more liquid and attractive its bonds are for investors. If the dominant currency country grows less than the rest of the world, its status as a safe haven erodes and interest rate differentials decline. This could explain the recent evidence of shrinking US return differentials on its cross-border bond portfolios.

Keywords: Dominant Currency, Safe Assets, US Dollar, Default

JEL classification: E42, F02, F33, N10

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NON-TECHNICAL SUMMARY

How does a country obtain the status of a dominant global currency provider, and how can it lose this position? This paper focuses on two key aspects: safety and size, and how they relate to each other. Both are necessary conditions for becoming a dominant global currency, in which a country enjoys an exorbitant privilege in the form of lower yields. This is the result of investors strongly preferring to hold bonds of the dominant country, thus willing to pay lower interest rates to the country with the dominant currency.

Our point of departure is that government bonds are not per se safe, as governments can default on their debt whenever a shock occurs that makes it tempting to do so. We argue that a larger economy helps to diversify this default risk. We discuss how regional and sectoral correlation of idiosyncratic shocks, as well the institutional features of the government are key to this diversification argument.

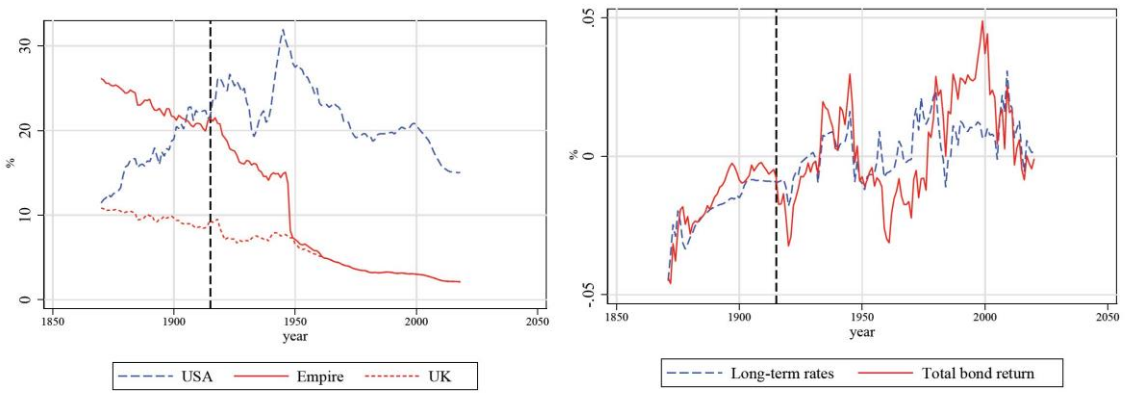
Next to safety itself, the overall size of the country also plays a role as a larger financial market makes it easier to facilitate transactions. This makes bonds of a large country more attractive, which means that there are interest rate spreads even in situations in which countries are comparably safe, but of different size. Furthermore, having safer government bonds also increases the debt capacity of a country, which allows for higher sustainable debt to GDP ratios. This amplifies the size effect and creates convenience yields.

Interest rate differentials are then composed of both a risk and a liquidity premium. While abundant supply of bonds is good from the perspective of liquidity provision, there is a trade-off as the country's risk premium increases at the same time. As in the Triffin dilemma, a country that acts as the global liquidity provider needs to be prudent not to overextend itself.

A previously dominant country can lose its exorbitant privilege if it is eventually overtaken by another rising country in terms of size and safety. The paper highlights several key factors that determine when a challenger could be ready to assume that role, and when it might not. The challenging country would need to have sufficient size, diversification, institutional quality and financial development.

Figure 1. Privilege of the US and the UK over time

(a) GDP shares (b) UK-US spread in USD



Note: World GDP share of the United States, the UK and the British Empire. The British Empire includes all countries and colonies for which the Empire has established control in a given year. Interest rates spreads in

USD are calculated the following way: You invest one USD in UK bonds in Pounds to exchange rates in period t and receive the interest rate in period t+1 in Pound, then exchanging those back to USD the current exchange rate. This is compared to US long-term bond interest rate payments in US Dollar adjusted for inflation. Total bond return is calculated by dividing coupon payments in period t plus the price for the bond in t by its price from the previous period. Spreads are very volatile, we apply a quarter-centennial moving average to smooth out short-run fluctuations. Source: Jorda et al. (2019), Bolt et al. (2018) and own calculations.

Privilège perdu ? L'ascension et la chute d'une monnaie mondiale dominante

RÉSUMÉ

Comment un pays obtient-il le statut de valeur refuge pour ses actifs et une monnaie mondiale dominante? Ce document de travail affirme que la taille est importante : plus un pays est grand et diversifié, moins le processus de choc sous-jacent de l'économie est variable. Les chocs susceptibles de conduire un gouvernement au défaut deviennent moins probables, ce qui implique un risque de défaut plus faible, des taux d'intérêt plus bas et un ratio dette/PIB plus élevé. En outre, plus la part d'un pays dans l'offre d'actifs sûrs mondiaux est importante, plus ses obligations sont liquides et attrayantes pour les investisseurs. Si la croissance du pays à monnaie dominante est inférieure à celle du reste du monde, son statut de valeur refuge s'érode et les écarts de taux d'intérêt diminuent. Cela pourrait expliquer la diminution récente des écarts de rendement aux États-Unis entre leurs actifs et leur passif transfrontalières.

Mots-clés : monnaie dominante, actifs sûrs, dollar américain, défaut de paiement

1 Introduction

The United States and the US Dollar play a crucial role in the international financial system. Dollar-denominated US government bonds are used extensively as safe assets by investors around the world. This special demand can give rise to yield differentials, lowering the required interest rates the US needs to pay when compared to other countries. This became known as the US' *exorbitant privilege*, a term coined in the 1960s by Valérie Giscard d'Estaing, at the time the French Minister of Finance, when discussing the position of the USD in the Bretton Woods system.

More recently, this expression has often been applied to return differentials between the US investment position compared to the rest of the world. Gourinchas and Rey (2007), Maggiori (2017) and Gourinchas and Rey (2022) describe how the US ends up holding riskier foreign assets than the rest of the world, who in turn holds more safe US assets. This implies that the US earns a higher return in normal times ('exorbitant privilege') while suffering losses in crisis (exorbitant duty').

Some recent papers argue that this privilege has waned or even disappeared since the Great Financial Crisis. Jiang et al. (2022) show that the US ex-post return differential was no longer positive from 2010 to 2019, and similarly Atkeson et al. (2022) demonstrate that the US Net Foreign Asset (NFA) position has substantially deteriorated. The debate is still open whether the recent ex-post differentials were due to a shift in expected return differentials, a higher than expected frequency of crisis, or even a bit of both. Here, we go back to the classic meaning of the expression and focus on the cost of debt and demand for liquid safe assets.

Motivated by that, this paper asks how a country might obtain the position of a global safe haven in the first place, how it might eventually lose that position and how safety is linked to convenience yields. We argue that both safety and size are necessary conditions for becoming a dominant currency, but also that these two properties are inherently connected. The larger the economy, the more diversified it will tend to be and so, *ceteris paribus*, the degree of safety will rise with size. This safety coincides with lower interest rates and also a larger debt capacity, reflected in a higher debt-to-GDP ratio. We also highlight two channels how convenience yields arise between two countries. First, safety is important because a defaulting bond has little to no value when financing investment. Second, the larger the country, the deeper the market for its bonds, which increases its liquidity and the probability that investors match with lenders in order to finance their investment opportunities. Finally, we show how rate differentials between two countries decline as the smaller country converges to the larger one, eventually leading to a loss of the convenience yield privilege when both countries are equally large and safe.

To obtain these answers, we develop a model in which a government defaults on its debt obligations whenever stochastic default costs are low and sovereign default becomes preferable to repayment. Investors anticipate this possibility and demand higher or lower interest rates depending on the distribution of such shocks. In larger economies[1], idiosyncratic shocks originate from more and more sources and therefore become less granular and less important. This is equivalent to a diversification of risk. A large economy with a low shock correlation across regions enjoys a lower aggregate shock variation. This decreases the likelihood of an aggregate tail-event that could lead to a default. We also formalize how these idiosyncratic shocks are aggregated. The aggregation depends on how much the nation-wide government cares about inter-regional smoothing[2]. If the government only considers the average national cost of default and focuses on the weighted average of regions (or sectors) for its default decision, the overall relative variation becomes smaller as long as those shocks are not perfectly correlated. This decreases the default probability as the economy grows. If the government also considers the distribution within the country and puts more weight on regions (or sectors) that would be hit hardest by a default, the default probability decreases even faster as the economy grows. This could explain how similarly sized countries have very different default probabilities and interest rates, reflecting differences in institutional setups or how shocks transmit through the economy. We also emphasize the degree of diversification for the default probability. The less correlated shocks across regions or sectors are, the stronger is the decline in overall volatility when the economy grows. In that sense, an optimal debt area in our setup is characterized by low shock correlation.

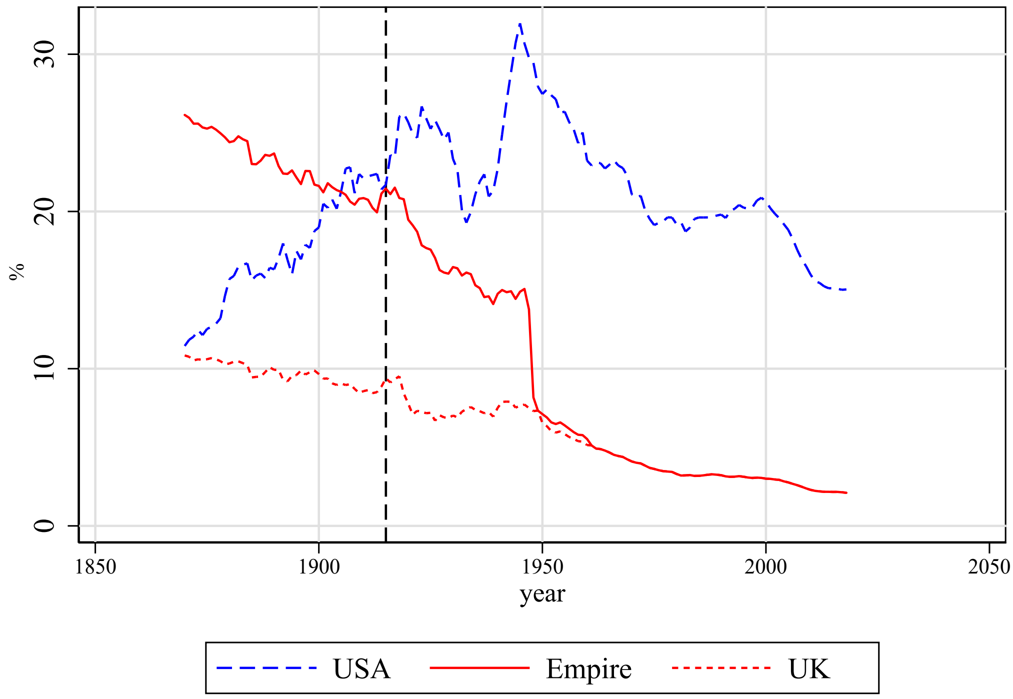
We embed this framework in a two-country open economy model in which investors can save in bonds of two countries and governments behave strategically. We show that the relative size of both countries matters for interest rate spreads. As the small country grows, default becomes less likely, its interest rate falls and debt-to-GDP rises, while the bigger country's interest rate increases slightly. We also introduce investment opportunities that are easier to facilitate the safer the bond and the larger the financial market is. This introduces interest rate spreads beyond pure risk premia. Depending on the long-term growth difference between countries, the original safe haven preserves its privilege if it remains the larger economy, but can lose it if it is overtaken by the other country.

[1]In terms of number or regions or a larger number of relevant sectors.

[2]This can also be interpreted as a proxy for institutional quality and its ability to provide inter-regional smoothing during sovereign default.

Historical Evidence There are not many instances where a country loses its position as the global dominant currency issuer, but one well known example is the USD taking the spot of the British Sterling, as the British Empire waned. In Figure 1, we highlight the historical evolution of the share of world GDP of the United States and the British Empire, consisting out of the United Kingdom and all of its colonies. Towards the end of the 19th century and the beginning of the 20th century, the US share of world GDP grew substantially, eventually overtaking the British Empire and becoming the world's largest economy at around 1915 (vertical dashed line).

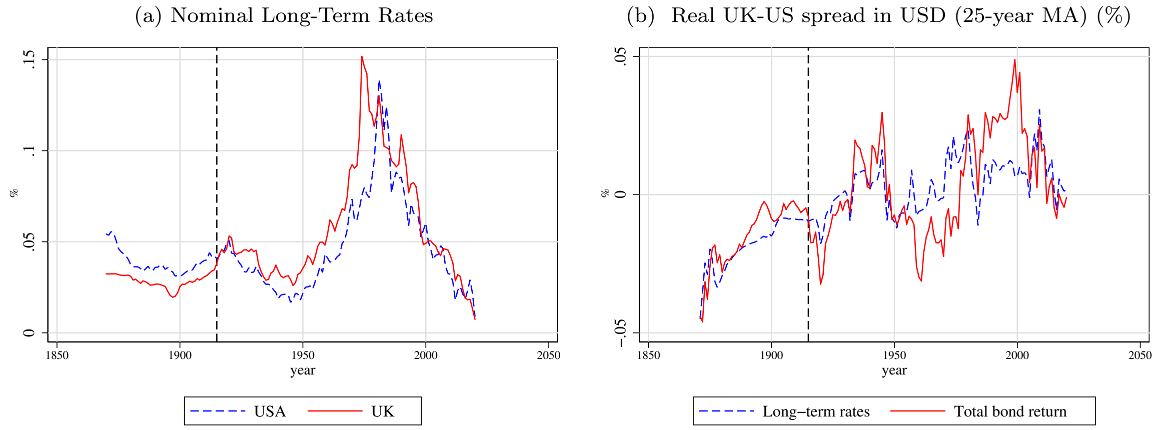
Figure 1: Share of world GDP



World GDP share of the United States, the UK and the British Empire. The British Empire includes all countries and colonies for which the Empire has established control in a given year. *Source:* Bolt et al. (2018) and own calculations.

In Figure 2 we document that this change in the position as the world's largest economy also coincided with a systematic change in interest rates that both central governments in Washington and London had to pay. As the left panel shows, the long-term interest rates of 10 year government bonds were higher for the United States throughout the period before 1915. After World War 1, the role reversed and the US was the country that paid lower interest rates. For a better comparison, we also look at it from the perspective of a global investor and construct the 'carry trade' time series of ex-post returns in USD, for both long-term interest rates and for the total bond return. The UK-US spreads are initially negative, indicating the presence of consistent UIP deviations that allow the UK cheaper access to funding relative to the United States. These spreads decrease gradually, eventually turning positive in the 20s and remaining generally positive up to recent times.

Figure 2: Interest rates



Note: Nominal long-term interest rates (10 year) of the US and the UK. Interest rates spreads in USD are calculated the following way: You invest one USD in UK bonds in Pounds to exchange rates in period t and receive the interest rate in period $t + 1$ in Pound, then exchanging those back to USD the current exchange rate. This is compared to US long-term bond interest rate payments in US Dollar adjusted for inflation. Total bond return is calculated by dividing coupon payments in period t plus the price for the bond in t by its price from the previous period. Spreads are very volatile, we apply a quarter-centennial moving average to smooth out short-run fluctuations. *Source*: Jordà et al. (2019) and own calculations.

In Table 1, we also show the means of long-term spreads in three periods, using several versions of long-term spreads. We consider both long term rates and total yearly bond returns, calculated in i) real terms (domestic CPI), ii) USD returns (ex-post return in USD) and iii) ex-post USD returns deflated by US CPI.[3] The evidence for an important shift after 1915 is present in all these measures, as is the waning of the privilege after the Great Financial Crisis. This is indicative of the privileged position the United States occupied for almost a century, as the global provider of safe assets and the USD as the dominant global currency. It's also apparent from the first two rows that considering the real return using the domestic CPI to deflate returns seems to generate a much smaller spread than a standard UIP comparison (both nominal or deflated by the US CPI). This points to an important trend of USD real appreciation relative to the UK, as the latter's CPI has not increased as much as the GBP has depreciated vis-a-vis the USD.

More recently, the difference between both countries eroded substantially. Since 2009 spreads are very close to zero across the board and no longer even positive when looking at implied yields rather than bond returns, reflecting the possibility that the US could be losing its exorbitant privilege, as argued by Atkeson et al. (2022) and Jiang et al. (2022). As we will argue in this paper, this is also related to the accelerated fall in the share of the United States in global GDP since the 2000s.

[3]We also exclude the war periods of 1914-1919 and 1939-1947 as in Jordà et al. (2019).

Table 1: UK-US long term interest spreads over time

Spread rate	1870-1914	1915-2008	2009-2020
Real long rate	-1.81%	0.13%	-0.24%
Real total bond return	-1.59%	0.17%	0.16%
USD long rate	-1.34%	1.29%	-0.11%
USD total bond returns	-1.13%	1.33%	0.07%
Real USD long rate	-1.36%	1.27%	-0.10%
Real USD total bond returns	-1.13%	1.32%	0.11%

Note: Yearly average spreads of long term interest rates between the United Kingdom and the United States, excluding the World War periods of 1914-1919 and 1939-1947. *Source*: Jordà et al. (2019) and own calculations.

Literature The literature that discusses the US' exorbitant privilege has mostly focused on return differentials between the international investment position of the US, when compared to the rest of the world vis-a-vis the US. Gourinchas and Rey (2007) highlighted that the net external position to the US was akin to being the "world banker", borrowing short and lending long. In Gourinchas and Rey (2022), the same authors describe how the US position has also an element of insurer of the rest of the world as it provides safe assets while holding risky assets. The return differentials then arise because of risk premia and the US provides wealth transfers to the rest of the world during crises (the 'exorbitant duty'). The exorbitant privilege is therefore akin to an insurance premium during normal times. A similar view is taken by Maggiori (2017) who shows that the more financially developed country will take the riskier investment positions and so should be short in debt and long in equities. We argue that the larger country is also safer because it is more diversified, leading to a lower probability of default for a given debt-to-GDP ratio. Moreover, its liquid safe assets can also easily be sold or used as collateral when investment opportunities arise, which can additionally increase spreads due to convenience yields.

Jiang et al. (2021) develop a model of the global financial cycle in which the US dollar plays a special role as a provider of safe assets. They assume that US bonds are safe and that convenience yields arise via money-in-utility. Our paper asks how such safety is generated in the first place and how this safety links to the emergence of convenience yields when compared to other foreign bonds. We therefore put the focus on the question why the US became the safe haven and overtook other countries.

Some recent papers argue that the privilege has waned or even disappeared in the years since the Great Financial Crisis. Jiang et al. (2022) show that the ex-post return differential was no longer positive for the US from 2010 to 2019, and similarly Atkeson et al. (2022) demonstrate that the US' NFA position has substantially deteriorated. Our paper contributes to this strand of literature by explicitly modelling why US (government) assets are considered to be more safe and how the US obtains (and might lose) this position in the first place. Vicquéry (2022) provides systematic accounts for the historical structure of the international financial system, describing how the British Empire was at the center of the international monetary system, but was then overtaken by the US. Eichengreen et al. (2017) document in a similar way the evolution of the international financial system, noting how several currencies can be at the same time relevant for the global economy during a transition phase. We explicitly model how such a transition happens and provide evidence for our main mechanism: The relative size of the dominant country's economy relative to the world.

Some papers have already argued the size of an economy (and its global trade share) are important for currency returns. Hassan (2013) shows that bonds of larger countries feature lower interest rates, as they insure against shocks that affect a larger part of the world economy. Therefore, larger countries would also be 'privileged' in the sense of low cost of debt. Our notion of size and safety are closely related to this idea. In a similar way, Richmond (2019) highlights the role of trade centrality of countries for their importance in the world economy. He also provides evidence that a larger world GDP share of a country reduces the currency excess returns which means that interest rates are lower. Ready et al. (2016) argue that countries differ in their exposure to global endowment shocks in the long-run. They measure this exposure for G7 countries, noting that these exposures could reflect different fundamentals. Their calibrated model can indeed replicate the differences in currency returns.

Hassan and Zhang (2021) discuss the macroeconomic origins of currency and country risks. They note that country-specific "risk-free rates" differ substantially between countries and that those countries with lower (safe) rates are also able to accumulate relatively more capital having higher capital-to-output ratios. Another strand of the literature emphasizes the role of convenience or liquidity yields for US government bonds. Krishnamurthy and Vissing-Jorgensen (2012) analyze the aggregate demand for treasury debt and find that the special status of US treasury in terms of liquidity and safety drive down its interest rate by on average 73 basis points between 1926 and 2008 with 46bp attributed to liquidity and 27bp to safety. This special status of US debt saves the government interest rate costs of about 0.25 % of its GDP, a fact that is also present in our analysis as higher diversification and liquidity allow for a larger debt capacity of the dominant global currency country.

What makes government debt safe? He et al. (2019) highlight strategic interactions between investors and the size of a country's debt market. Our paper focuses on the determination of a country's fundamentals as its size grows and strategic competition between governments' bond supply. We also allow for an endogenously changing aggregate amount of bonds, with governments taking into account both the impact on safety and liquidity. Similar to Chahrour and Valchev (2022) we emphasize the role of the overall size of an asset market. The larger the share of the bond in world markets, the easier it is to facilitate transactions.

Choi et al. (2022) note that the US are the most important provider of safe assets. They discuss the possibility that the US could use this position and exert market power over their asset supply. This results in under provision of US government bonds and a markup implying lower interest rates for the United States reflecting the exorbitant privilege. Through the lens of this model a decline in the US markup can be explained by an increase in safe assets (outside) the United States. This increase in competition leads to lower interest rates and a decline of the exorbitant privilege. We show that similar effects are present even without the US behaving as a monopolistic supplier. In our baseline model, as foreign bonds become safer, their interest rates decline and risk-averse investors become more willing to hold foreign bonds as well. This relative safety effect slightly increases US bond rates and erodes the US advantage as the main supplier of safe assets. In our paper, we consider also the impact of supply on market depth and liquidity and we explore the implications of market power (and under provision of safe assets) in section 4.2.

Bianchi et al. (2023) provide a theory why convenience yields arise and how they move the exchange rate. They argue that financial institutions have strong precautionary demand for (safe) US assets as they are subject to volatile financial flows. This additional demand gives rise to systematic convenience yields, also impacting the US dollar exchange rate. Given that other countries are also able to generate safe assets in the model, systematic US convenience yields require USD funding to be overall more volatile than funding in other currencies. This explains why demand for US safe assets is relatively higher than for other safe asset currencies where the funding is less volatile. We model the additional liquidity element of safe bonds as the ability to liquidate safe assets to finance investment opportunities, which in turn drives up demand for the safest bond and consequently lowers its interest rates.

Farhi and Maggiori (2018) develop a model of the international monetary system, in which a hegemon issues international reserves to the rest of the world. The hegemon has –as in our model– a lack of commitment and can implicitly default on its reserves via a depreciation of its currency. The stability of the monetary system is determined by how many reserves are issued by the hegemon, which ultimately determines the probability of a default. They also discuss how the system changes if competition arises by other reserve issuers. Our approach differs as it explicitly considers the size of the country as an important determinant for the default probability. Not only does the amount of debt relative to GDP matter for the default probability, but also the total amount of GDP. Furthermore, we emphasize that competition between countries can also happen in a situation with considerable difference in absolute safety, which increases spreads between countries even further.

2 Baseline Model

We consider an economy with 2 countries, Home (H) and Foreign (F). Each country is populated by one central government and investors. As standard in the endogenous default literature, we assume the governments are impatient relative to investors, providing a motive to borrow. Due to limited commitment, the government has the option to (costly) default in period 2.

The two countries can be asymmetric in terms of their size $N \in \{N^H, N^F\}$. Size is defined as the number of regions that the country consists of. For now regions are ex-ante identical within and across countries, later we will explore heterogeneity in region size across countries. The central government preferences aggregate consumption of all of its regions into account under constant elasticity of substitution. Default costs are stochastic and region-specific. Under a certain probability these costs are sufficiently low, such that the government prefers to default instead of paying back its debt. Investors anticipate this possibility and demand higher interest rates in order to be compensated for default risk. A larger country therefore is able to diversify the risks from default costs better and, for a given level of debt, it will have a lower (ex ante) probability of default. This in turn lowers its cost of debt, so larger countries will also have a higher debt capacity relative to their GDP. For simplicity, we assume that regions are ex-ante symmetric and the total mass of investors (and therefore their wealth) is proportional to country size N.

A closed-form solution is provided in Appendix A.1 for the single country case, with iid default costs. We derive the first order conditions and depict supply and demand functions for government bonds, in order to build up intuition. It is straightforward to show that, as expected, larger N leads to more diversification, implying lower costs of debt and larger debt capacity. In the main text here, we directly provide the two-country setup to focus on the international dimension of the problem.

2.1 Government

We assume that the central government receives utility from a nation-wide aggregate of government consumption G_t:

$$\mathbb{E}\left[\log(G_1) + \delta \log(G_2)\right]. \tag{1}$$

δ is the time discount factor and log preferences imply some degree of (constant relative) risk aversion and willingness for intertemporal consumption smoothing. G_t is an aggregator of government consumption in N equally large regions:

$$G_t = \left(\sum_j^N \omega_j^{\frac{1}{\theta}} g_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}}.$$

ω_j indicates the regional size, and θ is a measure of inter-regional consumption substitution and therefore affects how the government combines consumption across different regions. When $\theta \to \infty$, the government simply takes the average consumption across all regions, as if the consumption in one region is perfectly substitutable with consumption from another region. For lower values, consumption becomes more complementary, implying that consumption dispersion across regions will reduce the overall aggregator. $\theta = 1$ corresponds to a Cobb Douglas aggregator while the extreme case of $\theta \to 0$ gives the Leontief case, in which the government cares only about the region with the minimum consumption and with the highest default costs. θ could reflect either political preferences of the government for equality across regions or reflect the ability of (an implicit tax system) to redistribute, particularly during times of sovereign default. We refer to θ as the institutional features of the government.

Each region generates income Y_1 for the central government in period 1 and income Y_2 in period 2. To increase consumption per region above Y_1 in period 1, the government issues bonds B to Home and Foreign investors at price Q. Aggregate government revenues are then $QB + NY_1$ while each region j consumes

$$g_{1,j} = QB/N + Y_1$$

Every region therefore receives an equal share of funds[4]. In period 2, the government either pays back all the debt B to investors, or it partly defaults. In case of default, it pays back a *fixed* share κ of the original debt repayment to investors and it suffers region-specific random dead-weight losses of ψ_j of output Y_2 that cannot be recovered by the government or investors. Regional consumption g_2 of region j is then given by

$$g_{2,j} = \begin{cases} Y_2 - B/N & \text{if no default} \\ (1-\psi_j) \cdot Y_2 - \kappa B/N & \text{if default} \end{cases}$$

The government defaults if country wide consumption G_2 is bigger under default than under repayment, that is if

$$\left(\sum_j^N ((1-\psi_j)Y_2 - \kappa B/N)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta-1}{\theta}} > \left(\sum_j^N (Y_2 - B/N)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta-1}{\theta}} \tag{2}$$

Every region features an identical process for ψ with cross-regional correlation ρ.[5] For given values of the other parameters and variables, the inequality is satisfied with probability p. The (ex-ante) default probability is therefore a function of debt, bond prices, model parameters and country size: $p = p(B, Q, \theta, \kappa, \rho, N)$. As long as the regional processes are not perfectly correlated ($\rho \neq 1$), considering more regions makes the overall variability of the left hand side smaller. Therefore, the inequality is less likely to be satisfied with larger N.

2.1.1 Institutional Features θ

To highlight the role of θ we consider the two polar cases: Perfect substitutability and perfect complements.

$\theta \to \infty$ (perfect substitutability) For $\theta \to \infty$, the government defaults if the average default cost is lower than the haircut $1-\kappa$ on average repayment costs per region, that

[4]Given that regions are ex-ante symmetric and there are no costs to allocate debt across regions, this would be the optimal allocation for any finite value of θ and would also achieve the optimum (but not uniquely) when $\theta = \infty$.

[5]Later, we will calibrate the marginal probability distribution of ψ to be uniformly distributed between zero and an upper limit $\bar{u}$, with the cross-regional correlation given by a copula, but for now the results are more general. We also consider equally sized regions, which is why $\omega_j^{\frac{1}{\theta}}$ is a constant and drops out in the inequality.

is if

$$\bar{\psi} Y_2 < (1-\kappa)B/N.$$

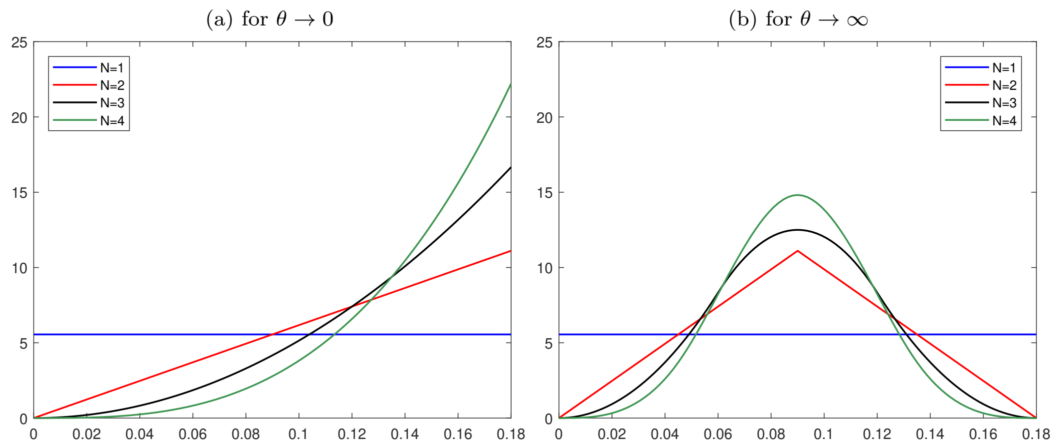
where $\bar{\psi} = \sum_j^N \frac{\psi_j}{N}$ is the average default cost. $\bar{\psi}$ has a probability distribution that is a function of N and so the inequality is satisfied with a certain probability (the default probability). If θ is uniform and iid, the relevant distribution of the sum is an Irwin-Hall distribution, see Appendix B.1.

$\theta \to 0$ (perfect complements) For $\theta \to 0$, the government only defaults, if the region with the largest default costs is still better off with a default, that is if

$$\max_j \{\psi_j Y_2\} < (1-\kappa)B/N.$$

For this case, the distribution function can also be computed directly when ψ_j are iid. The lower the θ – ceteris paribus – the less likely the government will be to default as the default cost probability distribution function features more and more mass on the right tail. That is, the more regions yo have the more likely it is that you have one region with high default costs. This prevents a government from defaulting, as it cares a lot about this region.

Figure 3: PDF of average aggregate default costs when $\rho = 0$



Note: Panel (a) shows the probability density function (PDF) of $\max\{\psi_i\}$ ($\theta \to 0$). Panel (b) shows the PDF of $\bar{\psi}$ ($\theta \to \infty$). Both are plotted for $N = \{1, 2, 3, 4\}$ where $\psi \sim Uni(0, 0.18)$ and iid.

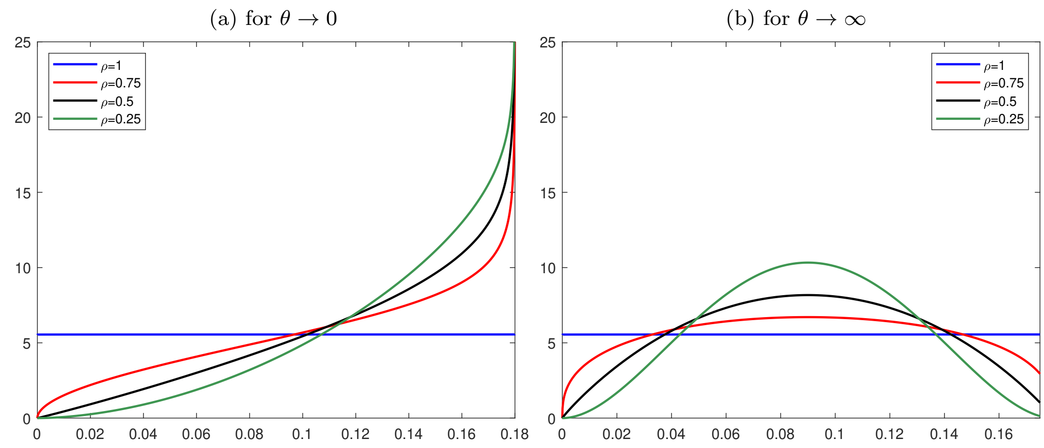
Figure 3 displays the probability distribution function of default costs for the two polar cases and varies the number of regions between $N = 1$ (blue line) and $N = 4$ (green line). In the case of perfect complementarity ($\theta \to 0$), the probability mass increasingly shifts to the right as the number of regions grow. It becomes increasingly likely that at least one region has very large default costs . This makes it less likely for the government to choose default if there are many regions. In the case of perfect substitutability ($\theta \to \infty$), the probability mass of the sum of uniforms increasingly centers around the mean. The more regions you have, the more certain a government is to have average default costs that are close to the mean. If the mean of default costs is large enough, this makes default also less likely the more regions there are.

2.1.2 Cross-sectional correlation of default costs ρ

We also consider different values for the correlation of shocks across regions to highlight the importance of diversification. This could be interpreted as how many different sectors an economy features or reflect a set of measures of diversity within a country.

We introduce correlation within regions by making use of Gaussian copulas. A Gaussian copula is a joint probability distribution function over the hypercube $[0,1]^N$ which can be mapped to the joint distribution of ψ^j for $j = 1, .., N$. One particular feature of Gaussian copulas is that marginal probability distributions remain uniform and it can be fully specified by a simple correlation matrix Ω. For simplicity, we set the non-diagonal elements of Ω to be all equal to ρ, so we can characterize the copula with a single parameter.[6] Since marginal distributions are uniform, when $\rho = 0$ we have the same distribution as in the previous section, with each ψ_j being drawn from a iid uniform distribution.

Figure 4: PDF of average aggregate default costs, N=4



Note: Panel (a) shows the probability density function (PDF) of $\max\{\psi_i\}$ ($\theta \to 0$). Panel (b) shows the PDF of $\bar{\psi}$ ($\theta \to \inf$). Both are plotted for $N = 4$ where $\psi \sim Uni(0, 0.18)$ with varying correlations across regions ρ.

The higher the correlation across regions, the closer the distribution gets to the original uniform distribution. In the extreme case of perfect correlation (the horizontal blue line),

[6] Since Ω is a correlation matrix, diagonal elements are all equal to 1.

adding more regions does not alter the aggregate distribution, as all regions draw the same shock. As a consequence, the lower the correlation across regions, the more diversified an economy gets and the less likely will be a default of the government. In a sense, an *optimal debt area* requires low (or even negative) correlation of shocks across regions, as huge nationwide shocks become less likely the larger and more diverse the country is. Indeed, the literature has observed that interest rate spikes and sovereign default usually coincide with heavy recessions, see for example Arellano (2008), Neumeyer and Perri (2005) and Uribe and Yue (2006). In our setup more regions with low shock correlation create safer and less variable fundamentals for the nationwide government, increasing the debt capacity of the country.

2.1.3 Competition in Bond Supply

The government maximizes (1) subject to the aggregator and the payouts in each period and state. We consider two setups: one in which governments take prices and the default probabilities as given and another in which they know that the amount of bonds (of both governments) influences prices and default probabilities. In the case in which governments act as price takers, they do not internalize that an increase in their bond supply increases the probability of default. In the more realistic case of governments being price makers, they behave as monopolistic suppliers of their own bonds. In particular, both governments engage in a Cournot-style competition for funds in the international market.

We provide details on the government's optimization problem in Appendix B. The first order condition together with the probability of default determines the supply of bonds. The problem of the Foreign government is analogous, with all variables being denoted by a star *. As interest rates and default probabilities are endogenous objects in the case of price makers that depend on not only each country's choice, but also the other country, we solve the two period-model via an iterative value function iteration algorithm.

2.2 Investors

Each country is populated by a mass of investors proportional to the country's number of regions N. Investors have Epstein and Zin (1991) preferences

$$U = \left[(1-\beta) C_1^{1-\sigma} + \beta E \left(C_2^{1-\xi} \right)^{\frac{1-\sigma}{1-\xi}} \right]^{\frac{1}{1-\sigma}} \tag{3}$$

with a time-discount factor of $\beta < 1$. Epstein and Zin (1991) allow to distinguish between the willingness to smooth consumption over time (σ) and the risk aversion (ξ). These preferences nest the special cases of CRRA preferences if $\sigma = \xi$ with risk neutrality ($\sigma = \xi = 0$) and log preferences ($\sigma = \xi = 1$).

Consumption of a given regional investor in the Home country in period 1 is given by

$$C_1 = W_1 - QB_H/N - Q^* B_F/N \tag{4}$$

where B_H denotes H bonds held by investors in H and B_F are F bonds held by investors in H. Regional consumption for foreign investors is $C_1^* = W_1^* - QB_H^*/N^* - Q^* B_F^*/N^*$. In period 2 governments pay back their debts, or choose to default.

2.2.1 Investment Opportunities

Importantly, we also assume that in the beginning of the second period, with probability χ, an investor i gains access to an investment opportunity in country j. This investment can be financed with country-j specific bonds only *if and only if* country j is not in a default state. In that case, those with investment opportunities have the possibility to receive a return of R^+ on their (scalable) investment. The other mass of investors $(1-\chi)$ do not have direct access to these country-specific investment opportunities. However, they can lend their net worth to those that do. This lending procedure is subject to search and matching frictions. Financial intermediaries specialized in country-j bonds collect $\mathcal{B}_j = (1-\chi)B_j + (1-\chi)B_j^*$ bonds and match it to the mass of investors with investment opportunities $X = \chi(N + N^*)$. Following Chahrour and Valchev (2022), we assume the following matching function :

$$M_j = m \frac{\mathcal{B}_j X}{\mathcal{B}_j + X}$$

where the parameter m represents velocity of asset holdings, reflecting the number of transactions a certain amount of bond holdings can sustain. It therefore also reflects average credit duration. The probability of a successful matching is then given by

$$\mu_j = \frac{M_j}{X} = m \frac{\mathcal{B}_j}{\mathcal{B}_j + X}.$$

The match surplus R^+ is distributed via Nash bargaining, so a fraction ν goes to the borrower with investment opportunities while $1-\nu$ is allocated to lenders. The intermediaries are perfectly competitive and receive zero profits. Finally, investors have the possibility of absconding with a fraction of the borrowed funds. As in Kiyotaki and Moore (1997), the participation constraint introduces a borrowing limit. Given the standard nature of the friction, we do not model the full agent-principal problem and summarize it through a reduced form borrowing limit, which is a fraction ζ of the borrower's own funds b. The expected additional return R_j^I due to the existence of investment opportunities is

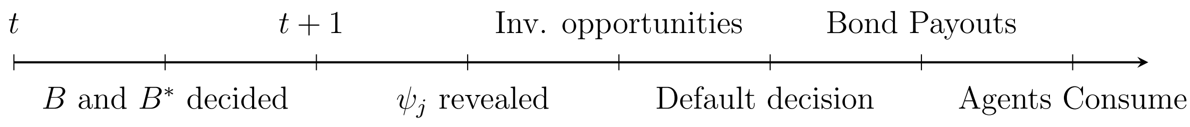
$$R_j^I = (1 - p_j)\left(\chi R^+ + \chi \mu_j \zeta \nu R^+ + (1 - \chi) R_j^D\right) \tag{5}$$

where R^D is the return earned from depositing funds with intermediaries.[7]. In this expression, the first term is the expected gain from getting an investment opportunity and investing his own funds. The second term reflects the gain from an investor with an investment opportunity if it matched with a lender. The last is the expected value of bond holders without opportunities that they get from depositing their funds with financial intermediaries.

The main difference between additional returns of home and foreign bonds (per unit) lies in the default probability p_j and the matching probability μ_j. The matching probability depends on the financial depth of the bonds market $\mathcal{B}_j$, which depends on the current market value of the respective bond. As in Chahrour and Valchev (2022), the larger the size of the government bond market, the tighter the matching function and therefore more likely a borrower or a lender will find a match. However, in our paper safety also plays a role. Only bonds that are not in a default state can be used to finance investment opportunities. Large and safer bond markets will therefore be privileged from the perspective that matching between borrowers and lenders is easier. This in turn generates larger expected returns for the corresponding asset. Lowering credit risk not only lowers risk premia, but also increases convenience yields.

2.2.2 Timing and Optimization

Figure 5: Timeline of baseline model



[7]Given intermediaries are perfectly competitive, this is simply the ratio of total fund earnings divided by the total deposited funds: $R_j^D = \frac{\mu\chi(1-\nu)\zeta R^+ \mathcal{B}_j}{(1-\chi)\mathcal{B}_j}$

The timing of the events is the following: Investors first decide how to invest in the bonds, knowing the default probability of both of them. Once the next period begins, default costs are revealed and it becomes public knowledge which government will default on their debt. After that each investors can have an investment opportunity with probability $1-\chi$ while the intermediation between those with and without investment opportunities takes place. The bonds are paid out (or defaulted upon) and finally agents consume whatever remaining wealth they have. An illustration of the timeline is shown in Figure 5. Overall, each of the two governments can either default or repay giving rise to four possible consumption states in period 2. Furthermore, the investment opportunity for no-default bonds and the intermediation procedure creates in total 16 possible states for the investors. The investor's problem is given by:

$$\max_{B_H, B_F} \left[(1-\beta) C_1^{1-\sigma} + \beta \left(\mathbb{E}_t C_2^{1-\xi} \right)^{\frac{1-\sigma}{1-\xi}} \right]^{\frac{1}{1-\sigma}} - \Lambda(B_H)$$

where $\Lambda(B_H, B_H^*) = \frac{\phi}{\lambda} \left(\frac{QB_H}{QB_H + Q^* B_H^*} - \bar{B} \right)^{\lambda}$ is a penalty term that lowers utility if investors hold less than a $\bar{B}$ share of domestic bonds. This term is symmetric for both countries – $\Lambda(B_F^*, B_F)$ – implying that this is not why an exorbitant privilege might arise. There are two reasons why we include such a term. First, we want to replicate the empirical pattern that investors tend to have a home bias in their portfolio. Second such a term helps to pin down investment positions, in particular in the case where investors are risk-neutral. It does not give rise to systematic convenience yields for one country, as both type of investors target bonds of their corresponding country.[8]

The investors optimize their portfolio, giving rise to first order conditions that shape their demand curve for bonds. The full set of optimization problems, together with the budget constraint and the first order conditions, are described in more detail in Appendix B.

2.3 Equilibrium

The equilibrium requires that the first order conditions of all investors and all governments are satisfied for a set of variables $\{B_H, B_H^*, B_F, B_F^*, Q, Q^*\}$. Furthermore, bond markets

[8]We will also calibrate ϕ to be a very low value, so that it pins down portfolios in the risk neutral case without significantly distorting aggregate demand for each type of bond

clear

$$B = B_H + B_H^* \quad \text{and} \quad B^* = B_F + B_F^*,$$

as well as goods markets and budget constraints for all states.

3 Calibration

This section provides a calibration for key parameters of the model. Table 2 shows the parameters, the corresponding calibrated values and their description.

Table 2: Calibration

	Value	Description
Default		
$\bar{u}$	0.187	Default costs upper bound, expected output lost 9.35%
κ	0.37	Average haircut of 37 %
Investors		
σ	0	Intertemporal substitution, no smoothing
ξ	0	Risk aversion, risk neutral
β	0.98	Time discount factor
W	1	Endowment in periods 1 and 2
$\bar{B}$	2/3	Domestic bond target $\approx 2/3$ home bias
λ	2	Quadratic disutility for deviating from bond target
ϕ	0.01	Strength of disutitliy for deviating from bond target
χ	0.6	Share investors with inv. opportunity
ν	0.8	Share investors in Nash bargaining
m	8	Credit duration as in Chahrour and Valchev (2022)
ζ	0.5	Borrowing limit
Government		
δ	0.8	Time discount factor, impatient government
Y	1	Endowments in period 1 and 2
θ	$\in [0, \infty)$	Regional smoothing parameter
ρ	$\in [0, 1]$	Regional shock correlation
N	7	Size of Home (dominant) country
N^*	$\in [2, 7]$	Size of Foreign country
R^+	0.02	Investment opportunity, additional 2% return

The upper part of the table calibrates properties in the default state. The output loss in the model in case of default is uniformly distributed between 0 and $\bar{u}$. We choose a value $\bar{u} = 0.187$ implying an expected output loss of 9.35%. This number is motivated by the literature, that finds an average output loss of 1.099% per year (Trebesch and Zabel, 2017) and an average duration of 8 years (Uribe and Schmitt-Grohé, 2017). These two numbers together imply a net present value of output losses (discounted at β) equal to 9.348%. The average haircut on investors in case of default is set to $\kappa = 0.37$, in line with estimates by Cruces and Trebesch (2013).

Next we calibrate parameters that govern the investors' behavior. For now, we calibrate investors to be risk neutral and to have no desire to smooth consumption intertemporally. The time discount factor for investors is set to $\beta = 0.98$ implying yearly real rates of 2% for safe assets without convenience yields. The penalty term is calibrated such that investors would target a 2/3 home bias for their corresponding domestic bond, in a world where there is no default risk and the government can smooth out its consumption. This is in line with US numbers from the Congressional Research Service (Labonte and Leubsdorf, 2022) who documents that 33% of all US government bonds are held by foreigners. Estimates for the home bias vary between countries, see Coeurdacier and Gourinchas (2016). Acharya et al. (2014) find a mean home share of 69.4%. A low value of $\phi = 0.01$ ensures that the penalty term does not dominate the overall investment decision, yet helps pin down the cross-country allocation in risk neutral settings, while $\lambda = 2$ implies convex costs from deviating domestic bond holdings from the target $\bar{B}$. Regarding investment opportunities for investors, we choose a value of 60%, indicating that 60% of investors will have a direct investment opportunity. At the same time, we calibrate the Nash bargaining parameter ν to 80% indicating that investors with opportunities retain the majority of additional surplus, while intermediaries receive only a small share.

The government's discount factor is set to $\delta = 0.8$, so that it is impatient and has a strong incentive to issue debts in the first period. Without any commitment problem, the government would aim for a debt-to-GDP ratio of around 10% to ensure marginal utility equalization across periods. We consider for now two polar cases for θ (perfect substitution and Leontieff) to highlight the role of inter-regional consumption smoothing for the default decision. The correlation of shocks between regions can be between 0 and one, in the baseline we choose a correlation of 0.3. We also vary the size of both countries, to explore the behaviour of interest rate spreads, debt capacity and probability of default as a function of N^*. Last, R^+ is calibrated such that it gives an additional return of 2%. Together with χ slightly above 50%, a non-zero default probability of bonds, investors expect to have 1% more returns on their bonds. The matching function parameter m, the distribution of returns between borrowers and lenders ν, and the probability χ are calibrated to achieve convenience yields of around 25 bp initially for the dominant country.

This is in line with Du et al. (2018) who find an average convenience yield of 26 bp for 3 year horizon bonds[9].

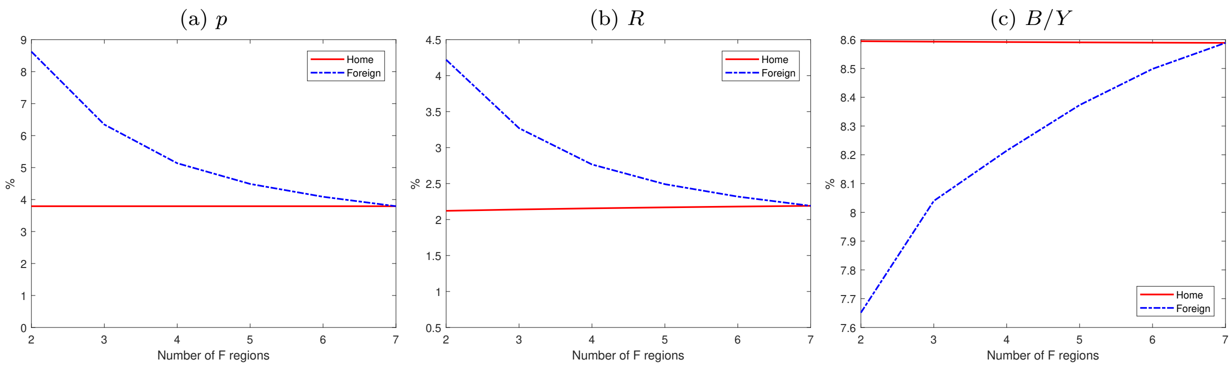
4 Results

In this section we highlight how key economic variables evolve when the Foreign country grows while the original dominant currency country (Home) remains at the same size. The first subsection demonstrates how the Home country loses its privilege as the Foreign country becomes as big as the Home country. We then discuss the role of bond competition between countries and in how far regional size differences matter, when countries are similarly safe.

4.1 Privilege Lost

We set initially $N^* = 2$, and $N = 7$, so that there is an important size difference between the two countries, and then explore what would happen if Foreign was closer or equal in size to the Home country. We first highlight the cases when there is perfect substitution across regions ($\theta \to 0$) and perfect complementarity ($\theta \to \infty$), under risk neutral investors and Cournot competition in bond supply. We later look at what happens when we depart from those assumptions. Figure 6, displays the steady state values for default probability, interest rates and Debt-to-GDP ratio as a function of F's size N^* under perfect substitution.

Figure 6: Main variables under perfect regional substitution

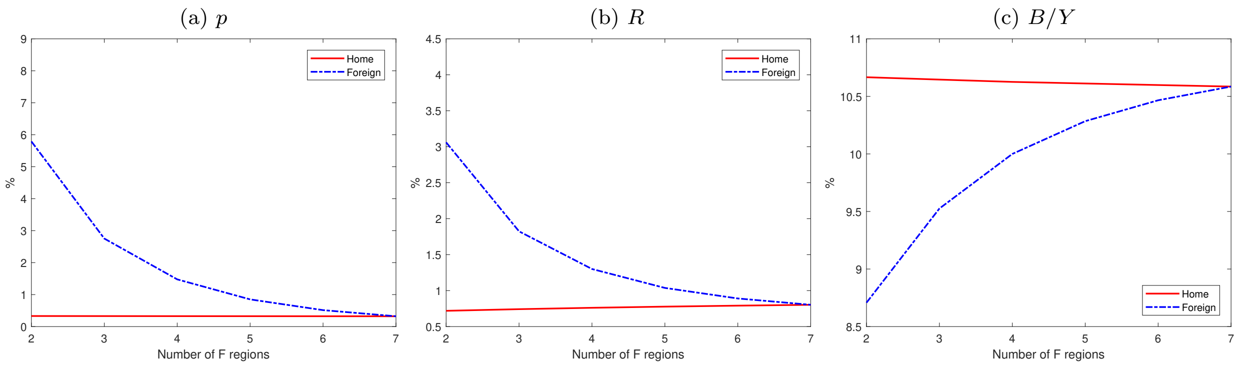


Note: Steady state values of key variables for Home (solid red) and Foreign (dashed blue) as a function of N^*, under Cournot competition, perfect substitution $\theta \to \infty$ and mild cross-sectional correlation $\rho = 0.3$.

[9]Other literature use different ways to measure convenience yields, resulting in different estimates. For example Krishnamurthy and Vissing-Jorgensen (2012) find slightly higher average US convenience yields for liquidity of 46 basis points. Our calibration is therefore on the conservative side.

In the left panel, we see that probability of default falls for F as N^* grows, and the debt-to-GDP ratio rises. As expected, when country F grows, so does its debt capacity and governments use this opportunity to raise additional funds. Probabilities of default fall as it's optimal for governments to benefit both in terms of quantities and prices, from the higher debt capacity. Eventually, when countries are exactly identical, then the privilege is no longer there and all variables equalize. Figure 7 shows the case of perfect complementarity, which is qualitatively very similar but features *higher* debt, and *lower* rates and probabilities of default for both countries. Since the country is less prone to defaulting due to its institutional features, the economy has a higher debt capacity, leading to a level shift in these variables.

Figure 7: Main variables under perfect regional complementarity



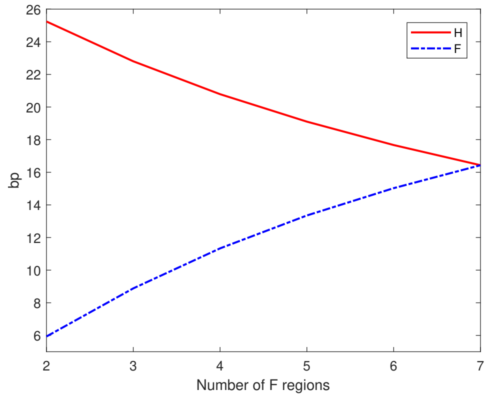
Note: Steady state values of key variables for Home (solid red) and Foreign (dashed blue) as a function of N^*, under Cournot competition, perfect complementarity $\theta \to 0$ and mild cross-sectional correlation $\rho = 0.3$.

Despite investors being risk-neutral, as N^* rises the H country rates also rise slightly, which is more noticeable for the case with perfect complementarity. This is due to the fact that the size of the H bond market is now a smaller share of total investor portfolios. This decreases the likelihood of a match per investor, reducing the liquidity value of H bonds. Therefore, the larger the dominant country is relative to the rest of the world, the higher its liquidity yields will be and the lower the cost of funding for the government. These liquidity yields are often described as convenience yields. We depict convenience yields as a function of N^* in Figure 8. These are defined as the expected gain from the ability to fund investment opportunities via the search and matching market:

$$R_j^{con} = (1 - p_j) \left(\chi \mu_j \zeta \nu R^+ + (1 - \chi) \frac{\mu \chi (1 - \nu) \zeta R^+ \mathcal{B}_j}{(1 - \chi) \mathcal{B}_j} \right)$$

Those include elements from equation (5) without the direct gains from investment opportunities. As discussed in the previous section, this expression is a function of liquidity (the matching probability μ_j) and the probability of default p_j. As F becomes safer, the

Figure 8: Convenience yields



Note: Steady-sate values of convenience yields for bond j as a function of N^* for Home (H solid red) and Foreign (F dashed blue). $\rho = 0.3, \theta \to 0$

fall in the probability of default improves convenience yields, but also liquidity improves as the country (and its debt market depth) grows. At the same time, the dominant country share of the world economy is falling, not only is it eroding its *relative* advantage with respect to F but also reducing the *absolute* value of convenience yield.

These two opposing forces highlight the dilemma faced by a dominant currency country. On one hand, the more of its bonds are held and traded across the world, the better its liquidity properties, but on the other hand it needs to be prudent in bond emissions so that its debt remains safe and therefore useful as a financing currency[10]. For the smaller country, relative growth improves both dimensions. It increases debt capacity, and with it better safety and liquidity services.

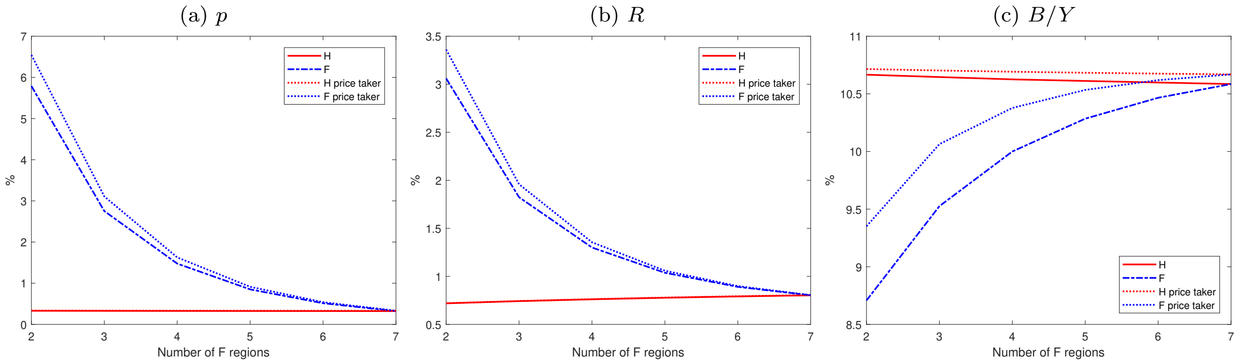
4.2 Strategic behaviour and the scarcity of safe assets

In our baseline model, as is standard in sovereign debt models with endogenous default, governments restrain their supply of bonds in order to reduce interest rates. At the same time, there is an opposite force in our model as governments have an incentive to increase the supply of bonds to improve its liquidity and increase the probability of a match. To examine the effects of strategic behaviour, we compare here with the case when governments do not consider the price impact of their actions and take bond prices as given.

Figure 9 compares the case of price taker governments (dotted lines) with the baseline Cournot competition. The right panel clearly shows that, as expected, debt-to-GDP would be significantly higher for F in the price-taker case as its government doesn't internalize

[10]This reflects elements of the traditional Triffin dilemma, Triffin (1961).

Figure 9: Price taker vs price maker, $\theta \to 0$



Note: Steady state values of key variabes for Home (solid red) and Foreign (dashed blue) as a function of N^*. Dashed lines show the case in which both are price takers. Both cases feature perfect complementarity $\theta \to 0$ and mild cross-sectional correlation $\rho = 0.3$.

the impact on the cost of funds. As a consequence, interest rates will be higher and so will probabilities of default. When the country is smaller, default probabilities (and therefore interest rates) are most elastic to quantities. As a result of the strategic behaviour, we then observe lower debt, lower default probabilities and lower interest rates. The same results hold for the case of perfect regional substituability, though both countries are in the region with more elastic quantities as the default probability remains large[11].

4.3 Size and diversification

In this section, we explore what happens when there is a cross-country difference in regional and investor endowments, but countries are equally diversified. Specifically we consider the following cases $0 < W^*/W = Y^*/Y \leq= 1$ and $N = N^*$. Although utility functions of governments and investors are homothetic, convenience yields introduce a role for size that introduces non-homothetiticies in the government problem under Cournot competition (via the matching function). To explore the role of size without introducing non-homothetiticity in the government problem, we consider here the case of price-taking governments which take liquidity as given. The role of size is then introduced through the market clearing but, conditional on liquidity, first order conditions are the same irrespective of size.

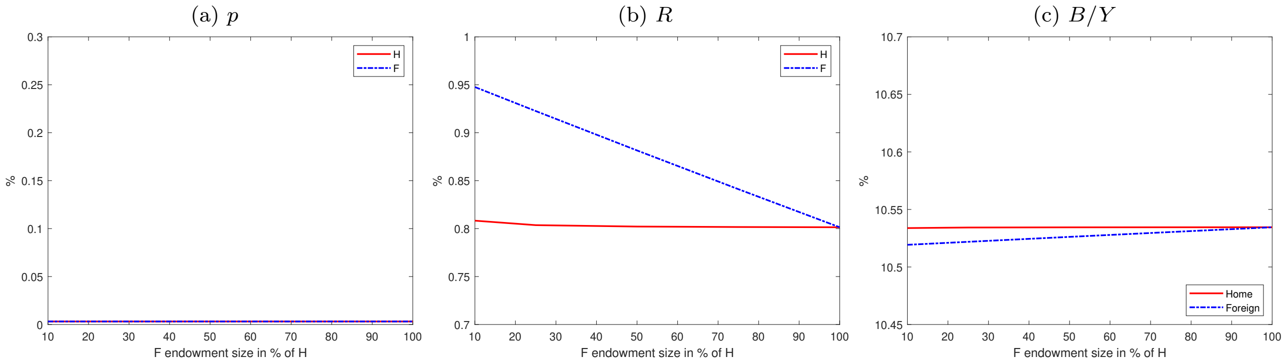
Figure 10 shows the main variables as a function of relative endowment size: $Y^*/Y = W^*/W$. Even though differences are much smaller, as in the case with $N^* < N$, there are still substantial interest rate spreads. As can be seen in the left graph, both countries are safe with very low probabilities of default. This means that the main difference in interest

[11]See Figure 12 in the Appendix.

rates are convenience yields, which are to the advantage of the larger country due to its superior liquidity properties ($B^*/B < 1$). The bigger the size difference, the larger is the spread between the two bonds.

The presence of convenience allows the larger country to have a higher debt capacity, leading to a slightly higher debt-to-GDP which in turn leads to a negligibly higher probability of default. This is because both countries are well-diversified and H features lower rates when $Y^* < Y$.

Figure 10: Relative endowment size and main variables



Note: Steady state values of key variables for Home (solid red) and Foreign (dashed blue) as a function of F's endowment in % of H. Both countries have the same number of regions (n), but differ in their endowment volume. Correlation between regions is $\rho = 0.3$, $\theta \to 0$. Governments are price-takers. All Y axis are scaled to have a range of 0.3%.

4.4 Risk-Averse Investors

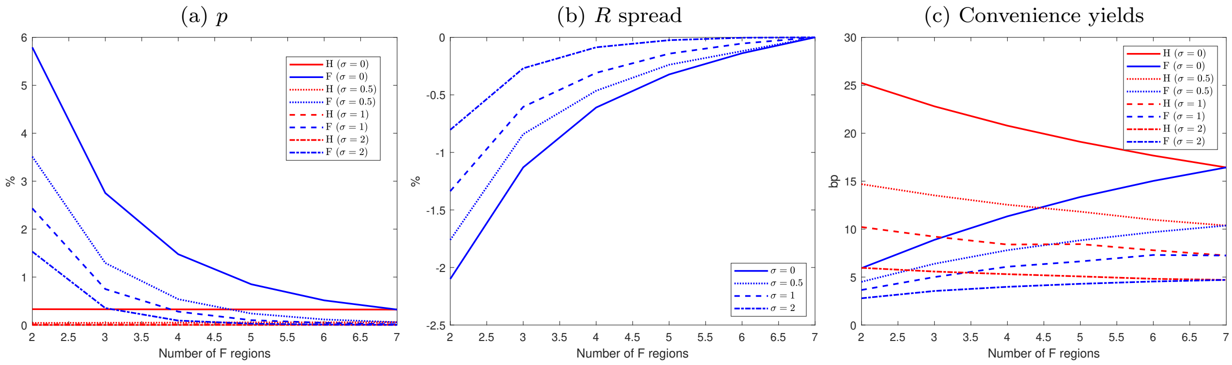
In this subsection we explore different values for risk aversion $\sigma = \{0, 0.5, 1, 2\}$, keeping the remaining calibration as before. Higher values of σ indicate that investors are more risk-averse. With larger risk aversion, the default probability of government debt falls, as investors are not willing to tolerate the risk of default. Governments reduce their Debt-to-GDP ratios in order to satisfy investors which now demand a higher return per unit of risk. Finite intertemporal elasticity of substitution means that they also demand a higher return in order to tilt their consumption profile[12].

The interest rate spread between countries therefore goes down, as the riskier country F reduces its default probability more in order to satisfy risk-averse investors. Convenience yields go down significantly, but more so for H. Even though a lower default probability increases convenience yields, the reduction in the size of the debt market dominates in this setup. This effect is stronger for H than F, because the latter reduces its default

[12] In the baseline we have risk neutrality and infinite intertemporal elasticity of substitution.

probability by more, implying that spreads are lower under higher risk aversion[13].

Figure 11: Risk-averse investors and main variables



Note: Steady state values of key variables for Home (red) and Foreign (blue) as a function of F's size. Correlation between regions is $\rho = 0.3$, $\theta \to 0$. Governments are price-makers. Different values for risk aversion σ other than neutrality are indicated by dashed or dotted lines.

4.5 Exorbitant Privilege and Exorbitant Duty

In period 1, country H runs a current account deficit as it takes on more debt with F investors, than the F government does with H investors ($B_F < B_H^*$). In period 2, countries decide to default or repay. If everyone repays, H's government pays back RB_H^* to F while H receives R^*B_F. In period 2, H generates a current account surplus, as its government pays back its debts. Nevertheless, individual investors in H enjoy excess returns as $R^* > R$, so our model also captures the definition of exorbitant privilege based on portfolio return differentials[14]. However, if there is a crisis in country F and its government defaults, it receives only a fraction of the payment $B_F R^*$ and there is a transfer of wealth from H to F. The exorbitant duty of the safe country during such a crisis, means that investors do not enjoy excess returns ex-post ($\kappa R^* < R$) in times of a bad shock for the Foreign country (or when there is a global crisis and both default). Although Gourinchas and Rey (2022) formulate the notion of privilege and duty through the heterogeneous portfolio composition of the US and the rest of the world, the presence of default risk can then generate similar type of dynamics even within the sovereign bond asset class when countries are not equally risky.

[13]However, it's possible that under a different calibration the increase in the price of risk reverses this results, despite the convergence in probabilities of default.

[14]However, note that we do not feature equity flows.

5 Conclusion

We develop a model of the international financial system in which governments can default on their outstanding debt implying that their assets are not automatically considered safe. How does a country evolve into a safe haven in such a setup? We argue that only if the government's economy is sufficiently large will default risk diversify enough so that the government bond can be considered as safe. We demonstrate in an open economy setup how one large country becomes the safe haven of the world economy, enjoying lower interest rates and convenience yields. However, its position can be challenged if other countries grow in size and their debt become safe as well. This leads to a gradual decline in interest rate spreads and eventually to a loss of the safe haven privilege as soon as the other country's economy becomes larger (and safer) than the incumbent safe haven country. Our model can explain how global interest rates on public debt declined while debt-to-GDP ratios were rising and US convenience yields were falling.

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A Appendix

A.1 Closed Form Solution for Closed Economy

There are two periods and two agents in the economy. Investors receive an endowment of W in both periods. In period 1, they can buy government bonds that pay interest rate R in period 2. Governments are more impatient than investors and face stochastic default costs in period 2. The government has the possibility of defaulting at period 1and is risk-averse. To obtain analytical solutions, we assume that in case of default investors receive a fixed share of the government's output, instead of bond repayments. Investors only take the expected utility in case of repayment into account when optimizing which allows -together with probabilities that are taken as given- for an analytical characterization of the problem. We compute for both risk-neutral and risk-averse investors.

A.1.1 Government

There is one risk-averse government that has the following preferences over consumption G_t:

$$\frac{G_1^{1-\gamma}}{1-\gamma} + \delta \frac{G_2^{1-\gamma}}{1-\gamma}$$

δ is the time discount factor and γ governs the degree of risk aversion. There is income $Y_1 = Y_2 = 1$ for the government in periods 1 and 2. To finance consumption in period 1, the government can issue bonds B to the investors. Consumption is then given by:

$$G_1 = B + Y_1$$

In period 2, the government either pays back all the debts with interest rate R to the investors, or it defaults and pays a penalty constituting a *fixed* share κ of income to investors and a *random* dead-weight loss of ψ that cannot be recovered by bondholders. The density function for ψ is denoted by $f(\psi)$, the cumulative density function by $F(\psi)$. Consumption G_2 and the payout to investors M are then given by

$$G_2 = \begin{cases} Y_2 - B \cdot R & \text{if no default} \\ (1-\kappa)(1-\psi) \cdot Y_2 & \text{if default} \end{cases} \qquad M = \begin{cases} B \cdot R & \text{if no default} \\ \kappa \cdot Y_2 & \text{if default} \end{cases}$$

The government defaults if the penalty is smaller than the overall repayment of debt:

$$Y_2 - B \cdot R < (1-\kappa)(1-\psi)Y_2 \tag{6}$$

The (ex-ante) default probability is $p = F\left((1-\kappa)\psi < \frac{B \cdot R}{Y_2} - \kappa\right)$. If ψ is uniformly distributed between a=0 and b=1, the probability is $p = \left(\frac{BR}{Y_2} - \kappa\right)/(1-\kappa)$

Assume log consumption ($\gamma = 1$) and that the government is a price taker and does not take into account that the amount of bonds issued influences the interest rates and the default probability. The government maximizes

$$\max_B \quad \log(Y_1 + B) + \delta\left((1-p)\log(Y_2 - BR) + p\left(\mathbb{E}_\psi \log(1-\kappa-\psi^e)Y_2\right)\right)$$

The first order condition for the (price-taker) government's problem is

$$\frac{1}{Y_1+B} - \delta\left(\frac{(1-p)R}{Y_2 - BR}\right) = 0 \tag{7}$$

Plug in p in the first order condition

$$\frac{1}{Y_1 + B} = \delta \frac{\left(1 - \frac{BR/Y_2 - \kappa}{1-\kappa}\right) R}{Y_2 - BR}$$

The function can be solved for R (also for B), it describes the supply of bonds. To simplify and obtain analytical solutions in the baseline, we set $\kappa = 0$ so that in case of default investors get no share of output. We consider the more general with $\kappa > 0$ in Section 2. The analytical supply function is characterized by:

$$R = \frac{\delta(Y_1 + B) + B \pm \sqrt{[\delta(Y_1 + B) + B]^2 - 4\delta(Y_1 + B)B}}{2\delta(Y_1 + B)B/Y_2}$$

A.1.2 Investors

There is a mass of investors who have preferences as

$$U = \frac{C_1^{1-\sigma}}{1-\sigma} + \beta\frac{C_2^{1-\sigma}}{1-\sigma},$$

where the time-discount factor is $\beta < 1$. The parameter that governs risk aversion is set to $\sigma = 0$ (investors are perfectly risk-neutral) and to $\sigma = 1$ (risk-averse preferences with log consumption). We begin with the risk neutral case and risk-averse investors with $\sigma = 1$ are considered later.

Risk Neutral Investors

$$U = C_1 + \beta C_2$$

Investors have a fixed endowment W_1 in period 1 and can either spend it on consumption or invest it in risky bonds B that are issued by the government:

$$W_1 = C_1 + B$$

In period 2 they receive endowment W_2 and consume their payouts from the bonds, denoted by M

$$C_2 = M + W_2$$

The payout M is uncertain as governments can default as described before.

Investors anticipate the possibility of default: The rate of return for risk-neutral investors needs to be so that they are indifferent between consuming everything today and investing everything. This means that the following equation needs to hold:

$$\beta \left(p\kappa Y_2 + (1-p)BR \right) = B \tag{8}$$

Depending on the functional form of the density function f, this can be analytically solved. With a uniform distribution between 0 and 1, $p = (\frac{BR}{Y_2} - \kappa)/(1-\kappa)$ and κ being a constant, the equation that needs to hold in equilibrium is:

$$\beta \left(\frac{\frac{BR}{Y_2} - \kappa}{1-\kappa} \kappa Y_2 + \left(1 - \frac{\frac{BR}{Y_2} - \kappa}{1-\kappa} \right) BR \right) = B$$

The solution to this problem for the bond demand function can be obtained analytically, again we set $\kappa = 0$:

$$R = \frac{1 \pm \sqrt{1^2 - 4\left(\frac{B}{\beta Y_2}\right)}}{2B/Y_2}$$

Risk-averse investors Investors maximize their expected lifetime consumption. They have log-preferences, i.e. $\sigma = 1$, they choose their individual bond holding b, aggregate bond holdings are given by B.

$$\max_b \quad \mathbb{E}\left[\log C_1 + \beta \log C_2\right]$$

$$\text{s.t.} \qquad C_1 = W_1 - b \quad C_2 = W_2 + \begin{cases} bR & \text{with probability} \quad 1-p \\ \frac{b}{B}\kappa Y_2 & \text{with probability} \quad p \end{cases}$$

Investors choose bond demand such that thy maximize their expected lifetime utility, they take interest rates, default probability and payouts in case of default as given:

$$\max_b \quad \log(W_1 - b) + \beta\left((1-p)\log(W_2 + bR) + p\log(W_2 + \frac{b}{B}\kappa Y_2)\right)$$

The first order condition gives bond demand as a function of the default probability:

$$\frac{1}{W_1 - b} = \beta\left(\frac{(1-p)R}{W_2 + bR} + \frac{p\frac{1}{B}\kappa Y_2}{W_2 + \frac{b}{B}\kappa Y_2}\right) \tag{9}$$

To obtain analytical results, we focus in a situation in which $\kappa = 0$, so that the second term on the right hand side does not play a role. Plug in p and note that in equilibrium we have $b = B$.

$$R = \frac{\beta(W_1 - B) - B \pm \sqrt{\left[\beta(W_1 - B) - B\right]^2 - 4\beta(W_1 - B)W_2 B/Y_2}}{2\beta(W_1 - B)B/Y_2}$$

Together with bonds supply from the government, equilibrium rates are determined.

A.2 Extension: United States

A.2.1 Government: Multiple regions

Consider a country (the US), that consists of two regions A and B. Both regions receive income Y_2 in the next period, the repayments of debts are split equally between regions. Both regions are also subject to a default cost process ψ_A and ψ_B. Both have the same distribution, but default can have different costs in each region ex post. The government consumption aggregator of these two equally sized regions in period t is:

$$G_t = \left(\omega^{\frac{1}{\theta}} G_{tA}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} G_{tB}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}}$$

where θ indicates how substitutable consumption for both regions and ω indicates the size (or the weight) of the region A in the country).

The government maximizes expected lifetime utility of the country:

$$\mathbb{E}\left[\log G_1 + \delta \log G_2\right]$$

The government defaults if consumption under default in period 2 is higher than consumption under repayment. Under equal weights $\omega = 1 - \omega$:

$$\left(((1-\psi_A)(1-\kappa)Y_2)^{\frac{\theta-1}{\theta}} + ((1-\psi_B)(1-\kappa)Y_2)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}} > \left((Y_2 - BR/2)^{\frac{\theta-1}{\theta}} + (Y_2 - BR/2)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}}$$

while in this case investors still receive $\kappa N Y_2$, where N is the number of regions. Looking at some particular cases:

$\theta \to \infty$

$$\begin{aligned}(1-\psi_A)(1-\kappa)Y_2 + ((1-\psi_B)(1-\kappa)Y_2 &> 2(Y_2 - BR/2)\\ \frac{1}{2}(\psi_A + \psi_B) &< \frac{1}{(1-\kappa)}\left(\frac{BR}{2Y_2} - \kappa\right)\end{aligned}$$

both goods are perfect substitutes, this basically means that the government simply sums up consumption in their objective function. Importantly the sum of these two identical processes features a lower variability than a single process. Lets call this process F_2. This makes (given same debt-to-GDP ratio) a default less likely for a given B as

$$F_2\left(\frac{(\psi_A + \psi_B)}{2} < \frac{1}{1-\kappa}\left(\frac{BR}{2Y_2} - \kappa\right)\right) < F\left(\psi < \frac{1}{1-\kappa}\left(\frac{BR}{Y_2} - \kappa\right)\right) \tag{10}$$

Generalizing for N regions we have

$$p = F_N\left(\frac{BR - N\kappa}{1-\kappa}\right) \tag{11}$$

where F_N is the cdf of the sum of N draws from a U[0,1] and follows therefore a Irwin-Hall distribution with parameter n=N (see section below)

$\theta \to 1$

$$
\begin{aligned}
(1-\psi_A)(1-\kappa)Y_2((1-\psi_B)(1-\kappa)Y_2 > (Y_2 - BR/2)^2 \qquad Y_2 = 1 \\
(1-\psi_A)(1-\psi_B) > \frac{(Y_2 - BR/2)^2}{((1-\kappa)Y_2)^2} \\
\psi_A\psi_B < \frac{(Y_2 - BR/2)^2}{((1-\kappa)Y_2)^2}
\end{aligned}
$$

The density for the product of two iid uniform is f(z)=-log(z), a lot of mass around zero that is quickly decreasing to low values around 1. Generalizing to N we have

$$
p = F_N\left(\frac{(Y_2 - BR/N)^N}{((1-\kappa)Y_2)^N}\right)
$$

where F_N is the cdf of the product of N independent uniforms. The pdf of the product of N uniforms [0,1] is as follows:

$$
f_N(x) = \frac{(-1)^{N-1}\log^{N-1}(x)}{(N-1)!}
$$

$\theta \to 0$

$$
\min\left((1-\psi_A-\kappa)Y_2, ((1-\psi_B-\kappa)Y_2\right) > Y_2 - BR/2
$$

The government only defaults, if both regions are better off defaulting. This logic extends to N regions. The government only defaults, if and only if all regions are better off.

Solve and get new probabilities of default given the process and given B and R. For example as $G_1A = B/2$ you arrive at $log(B/2 + B/2) = log(B)$ or you arrive at $log((B/2)^2) = 2 * (log(B) - log(2))$. maximization of log(B) and $2 * log(B)$ yields same result. Period 2 utility function is different and looks like this: $G_{2A} = Y_2 - BR/2$ or $Y_2(1-\kappa_A)$. Summing both up for perfect substitutability gives the objective function

$$
\begin{aligned}
&\max_B \quad \mathbb{E}\left[\log(G_{1A} + G_{1A}) + \delta\log(G_{2A} + G_{2B})\right] \\
&\text{s.t.} \quad G_{1A} + G_{1B} = B + 2Y_1 \\
&G_{2A} + G_{2B} = \begin{cases} 2Y_2 - B\cdot R & \text{if no default} \\ (1-\psi_A)(1-\kappa)Y_2 + (1-\psi_B)(1-\kappa)Y_2 & \text{if default} \end{cases}
\end{aligned}
$$

$$\frac{1}{B+2Y_1} = \frac{\delta(1-p)R}{2Y_2 - BR} \quad \text{same for } \theta = 1 \quad \frac{1}{B+2Y_1} = \frac{\delta(1-p)R}{2Y_2 - BR}$$

A.2.2 Investors

Investors are as before, the only difference is that we have a greater mass of investors that grows linearly with the number of regions. The overall endowment of investors in period 1 is then NW_1. Demand scales linearly with N in the FOCs, but the default probabilities in equilibrium and also the share of income they receive in case of default change.

Risk-neutral investors

$$\beta\left(p\kappa 2Y_2 + (1-p)BR\right) = B$$

$$\frac{1}{2W_1 - b} = \beta\left(\frac{(1-p)R}{2W_2 + bR} + \frac{p\frac{1}{B}\kappa 2Y_2}{2W_2 + \frac{b}{B}\kappa 2Y_2}\right)$$

Risk-averse investors

$$\max_b \quad 2\log(W_1 - b) + \beta\left((1-p)2\log(W_2 + bR) + p2\log(W_2 + \frac{1}{2}\kappa 2Y_2)\right)$$

The first order conditions are

$$\frac{2}{W_1 - b} = \beta\left(2\frac{(1-p)R}{W_2 + bR} + 2\frac{p\frac{1}{2}\kappa 2Y_2}{W_2 + \frac{1}{2}\kappa 2Y_2}\right)$$
$$\frac{1}{2W_1 - B} = \beta\left(\frac{(1-p)R}{2W_2 + BR} + \frac{p\kappa 2Y_2}{2W_2 + \kappa 2Y_2}\right)$$

for $\kappa = 0$ we have the analytical solution

$$\frac{1}{2W_1 - B} = \frac{\beta(1-p)R}{2W_2 + BR}$$

A.2.3 Generalization to N regions

$\theta \to \infty$ In general the sum of two independently distributed random variables X, Y is given by:

$$Z = X + Y$$

$$P(Z = z) = \sum_{k=-\infty}^{\infty} P(X = k)P(Y = z - k)$$

If we have continuous density functions f for X and g for Y, then this expression is

$$H(z) = \int_{\infty}^{\infty} F(z - t)g(t)dt$$

the convolution of the probability distribution is then

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x)f_Y(z - x)$$

The probability can therefore be computed for N regions. For the Uniform a generalization to N yields the Irwin-Hall distribution which is given by the following cdf:

$$F(x, N) = \frac{1}{2} + \frac{1}{2N!} \sum_{k=0}^{N} (-1)^k \binom{N}{k} (x - k)^N sgn(x - k)$$

with

$$p = F\left(\frac{BR - N\kappa}{1 - \kappa}\right) \tag{12}$$

$\theta \to 1$ For N regions we have

$$\prod_{i}^{N} \psi_i < \frac{(Y_2 - BR/N)^N}{((1 - \kappa)Y_2)^N}$$

Let the product of N uniform distributions be Z, we then have

$$f_N(z) = \frac{(-\ln z)^{N-1}}{(N - 1)!}$$

B Derivations Open Economy

The derivation of first order conditions is provided here.

B.1 Derivation Government: Intertemporal problem and default probability

We consider the government problem in which the share that goes to investors in case of default increases the more the government has issued bonds. We assume that investors receive a share κ BR, instead of a fixed share of output, while the government suffers a random default costs on output that cannot be recovered.

$$\max_B \log(G_1) + \delta\left[(1-p)\log(G_2(rep)) + p\mathbb{E}_{\psi<\tilde{\psi}}\log G_2(def)\right]$$

for $\theta \to \infty$ a government defaults if

$$NY_2 - BR < \sum_i^N (1-\psi_i)Y_2 - \kappa BR$$
$$\sum_i^N \psi/N = \tilde{\psi} < \frac{(1-\kappa)BR}{NY_2}$$

for a given density function f, the probability of default is then a function of B and R and is $p = f(\psi < \tilde{\psi})$. If f in uniformly distributed as in the example of the main text, the sum of uniformly distributed variables follows an Irwin-Hall distribution.

for $\theta \to 1$

$$\prod_i^N (Y_2 - BR/N) < \prod_i^N ((1-\psi_i)Y_2 - \kappa BR)$$

implicit function

for $\theta \to 0$

$G_t \to \min g_{it}$ default if

$$\min(g_{it}(rep)) < \min\{g_{it}(def)\}$$
$$\min\{Y_2 - BR/N\} < \min\{(1-\psi_i)Y_2 - \kappa BR/N\}$$
$$\max\{\psi_i\} < \frac{(1-\kappa)BR}{NY_2}$$

for iid uniforms the density function of the maximum value is

$$F(y) = \begin{cases} \left(\frac{y}{b-a}\right)^n & : y \in (a,b) \\ 0 & : y < a \\ 1 & : y > b \end{cases}$$

$$f(y) = \begin{cases} \frac{n}{b-a}\left(\frac{y}{b-a}\right)^{n-1} & : y \in (a,b) \\ 0 & : \text{ otherwise} \end{cases}$$

for $\theta \to \infty$ (additive) the government's first order condition is

$$\frac{1}{NY_1 + B} - \delta\left[(1-p)\frac{R}{NY_2 - BR} + p\mathbb{E}_{\psi<\tilde{\psi}}\frac{\kappa R}{\sum_i^N (1-\psi_i)Y_2 - \kappa BR}\right] = 0$$

which is

$$\frac{1}{NY_1 + B} - \delta\left[(1-p)\frac{R}{NY_2 - BR} + p\int_0^{N\tilde{\psi}} \frac{f(\psi^{\Sigma})}{F(\tilde{\psi})}\frac{\kappa R}{NY_2 - \psi^{\Sigma}Y_2 - \kappa BR}d\psi^{\Sigma}\right] = 0$$

for $\theta \to 0$ (Leontief) the government's first order condition is

$$\frac{1/N}{Y_1 + B/N} - \delta\left[(1-p)\frac{R/N}{Y_2 - BR/N} + p\mathbb{E}_{\psi<\tilde{\psi}}\frac{\kappa R/N}{\min\{(1-\psi_i)Y_2 - \kappa BR/N\}}\right] = 0$$

which is

$$\frac{1/N}{Y_1 + B/N} - \delta\left[(1-p)\frac{R/N}{Y_2 - BR/N} + p\int_0^{\tilde{\psi}} \frac{f(\psi)}{F(\tilde{\psi})}\frac{\kappa R/N}{\min\{(1-\psi_i)Y_2 - \kappa BR/N\}}d\psi\right] = 0$$

B.2 Value Function Iteration

We proceed by backward induction.

B.2.1 Period 2: Government Default condition

The government repayment condition is unchanged.

$$\left(\sum_{j}^{N}((1-\psi_j)Y_2-\kappa BR/N)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}} > \left(\sum_{j}^{N}(Y_2-BR/N)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}} \tag{13}$$

Polar cases are simply the mean or the min of loss compared to repayment.

B.2.2 Period 2: Investors

At the final period, investors consume all resources left. $2^3 = 8$ possible cases for $\left(R_{t+1}, R^*_{t+1}, R^+\right)$. In expectation:

$$\mathbb{E}(V_2) = E_1(V_2|R, R^*, B, B^*, b, b^f) = \sum_{s=1}^{8}\pi_s(W_s + b^+R^+ + R_sb + R^*_sb^f)^{1-\xi} \tag{14}$$

with $b^+ = b$ when $N > N^*$, $b^+ = b^f$ when $N < N^*$ and $b^+ = b + b^*$ otherwise. Note that $\pi_s = f(B, B^*)$, which will be important later but can also be seen in (13). There is no maximization to be done in period 2 so (R, R^*, B, B^*, b, b^*) are all states.

B.2.3 Period 1: Investors

In the first period investors decide how much to buy taking into account prices and aggregate quantities.

$$V_1(R, R^*, B, B^*, b, b^f, C_1) = \max_{b,b^f}\left((1-\beta)C_1^{1-\sigma} + \beta\left(\mathbb{E}(V_2)^{1-\xi}\right)^{\frac{1-\sigma}{1-\xi}}\right)^{\frac{1}{1-\sigma}} - \Lambda(b-\bar{b}) \tag{15}$$

subject to:

$$C_1 = W_1 - b^f - b$$
$$\Lambda(b-\bar{b}) = \frac{\phi}{\lambda}(b-\bar{b})^\lambda$$

with and noting again that $\mathbb{E}(V_2)$ is a function of (R, R^*, B, B^*, b, b^*).

B.2.4 Period 1: Government

$$V^{next} = \max\{V^N, V^D\}, \quad \text{where}$$

$$V^N = \log\left(NY_2 - BR\right), \quad V^D = \log\left(\sum_i^N \left[(1-\psi_i)Y_2 - BR/N(\kappa)\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}$$

The overall value function can explicitly be written as

$$V = \log\left(NY_1 + B\right) + \underbrace{\delta \mathbb{E} V^{next}}_{\delta(p\mathbb{E}[V^D] + \delta(1-p)\log(NY_2 - BR)}$$

the value function of default is

$$\mathbb{E}[V^D] = \int_0^{\psi_\theta} \log(G_2(\psi_\theta))dF(\psi_\theta)$$

If $\theta \to \infty$: $G_2 = (N - \psi^{sum})Y_2 - BR(\kappa)$

If $\theta \to 0$: $G_2 = (1 - \psi^{max})Y_2 - BR/N(\kappa)$

B.2.5 Period 1: Market clearing

Note that foreign investors are symmetric to local ones. Market clears when supply equals demand

$$B = Nb + N^* b^* \tag{16}$$

$$B^* = Nb^f + N^* b^{f*} \tag{17}$$

B.2.6 Pseudo-algorithm

We have V_2 analytically and can therefore maximize V1 numerically without grid methods (using fmincon to minimize $-V_1$, for example). We will go with grid methods here for the sake of it. Steps:

1. Generate grids for b, b^f, b^*, b^{f*}
2. Guess a solution for B and B^*
3. Recover R and R^* from supply curve

4. Try all grid points, pick the max V_1 and V_1^* for both countries

5. Check market clearing, compare B and B^* with guess and adjust solution

6. Repeat from 3 until convergence

B.2.7 Pseudo-algorithm government

1. Generate grids for b, b^f, b^*, b^{f*}

2. Guess a solution for B and B^*, use market clearing together with investors foc

3. Recover R and R^* from demand curve

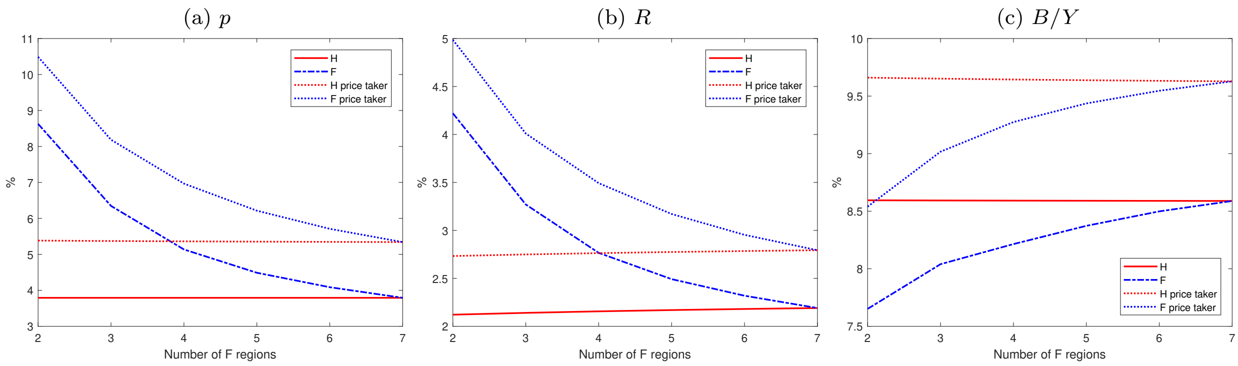
4. Try all grid points, pick the max V_1 and V_1^* for both countries

5. Check market clearing, compare B and B^* with guess and adjust solution

6. Repeat from 3 until convergence

B.3 Additional Figures

Figure 12: Price taker vs price maker, $\theta \to \inf$



Note: Steady state values of key variabes for Home (solid red) and Foreign (dashed blue) as a function of N^*. Dashed lines show the case in which both are price takers. Both cases feature perfect complementarity $\theta \to \inf$ and mild cross-sectional correlation $\rho = 0.3$. .